

Name KEY

Student ID _____

Check your (CRN) section number:

61082 8:00AM - 9:15AM

61084 9:30AM - 10:45AM

To get credit you must show work.

Page	Possible Points	Points Scored
1	12	
2	14	
3	14	
4	14	
5	12	
6	12	
7	10	
8	12	
TOTAL	100	

Scaled	
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1. [12 pts] A transverse wave traveling on a string is described by the expression

$$y(x,t) = 2.2 \text{ mm} \sin \{ 2\pi \left(\left(\frac{x}{1.2 \text{ m}} \right) + \left(\frac{t}{0.04 \text{ s}} \right) \right) \}$$

For the wave find:

- a) (2) The frequency, f

$$\frac{2\pi t}{0.04 \text{ s}} = 2\pi f t \Rightarrow f = \frac{1}{0.04 \text{ s}} =$$

$$f = 25 \text{ Hz}$$

- b) (2) The Wavelength, λ

$$\frac{2\pi}{1.2 \text{ m}} x = \frac{2\pi}{\lambda} x \Rightarrow \lambda = 1.2 \text{ m}$$

$$\lambda = 1.2 \text{ m}$$

- c) (4) The speed, v , and the direction of the wave

$$v = \lambda f = (1.2 \text{ m}) \left(25 \frac{1}{\text{s}} \right) = 30 \text{ m/s}$$

$$v = 30 \text{ m/s}$$

direction $-X$

- d) (4) The transverse velocity, v_y , of the string at $x=0.6 \text{ m}$ and $t=0$.

$$y = A \sin(kx + \omega t)$$

$$\frac{\partial y}{\partial t} = \omega A \cos(kx + \omega t)$$

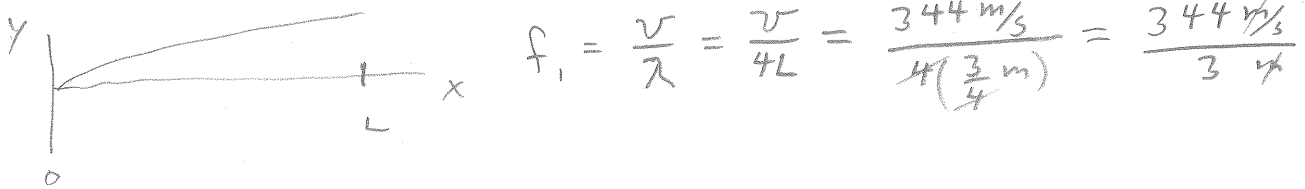
$$kx + \omega t \Big|_{\substack{x=0.6 \text{ m} \\ t=0}} = 2\pi \left(\frac{0.6 \text{ m}}{1.2 \text{ m}} \right) + 0 = \pi$$

$$v_y = -0.356 \text{ m/s}$$

$$v_y = \frac{\partial y}{\partial t} = \left(\frac{2\pi}{0.04 \text{ s}} \right) 2.2 \text{ mm} \cos(\pi) = -345.575 \frac{\text{mm}}{\text{s}} \approx -0.356 \text{ m/s}$$

2. [14 pts] A pipe of length $L=0.75\text{m}$ is closed at one end and open at the other end. Use $v=344\text{ m/s}$ for the speed of sound.

a) (3) Find the frequency of the fundamental, f_1 .



$f_1 = 114\text{ Hz}$

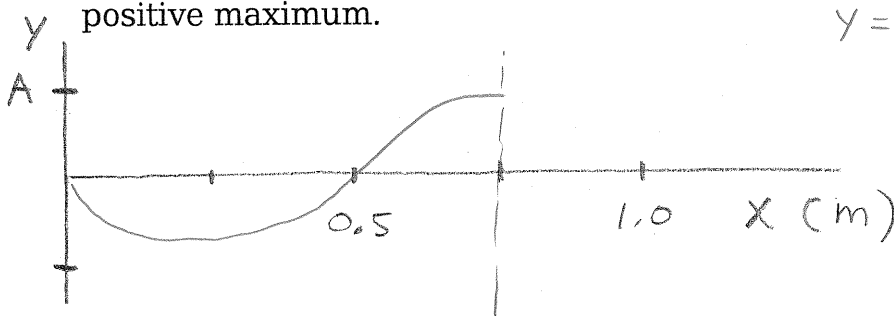
b) (3) Find the frequency of the first overtone, f_3 .



$$f_3 = 3f_1 = 3\left(\frac{344\text{ m/s}}{3\text{ m}}\right) =$$

$f_3 = 344\text{ Hz}$

c) (4) Sketch the displacement, $y(x)$, for the air in the pipe for the first overtone when the displacement at the open end of the pipe is a positive maximum.

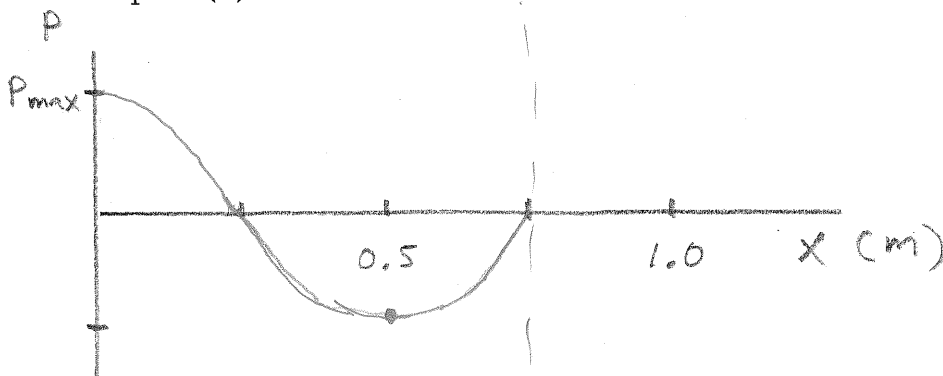


$$y = -A \sin_{sw}(kx) \sin(\omega t)$$

$$\frac{\partial y}{\partial x} = -kA \cos(kx) \sin(\omega t)$$

No need to show this

d) (4) Sketch the pressure change, $p(x)$, in the pipe at the same time as in part (c).



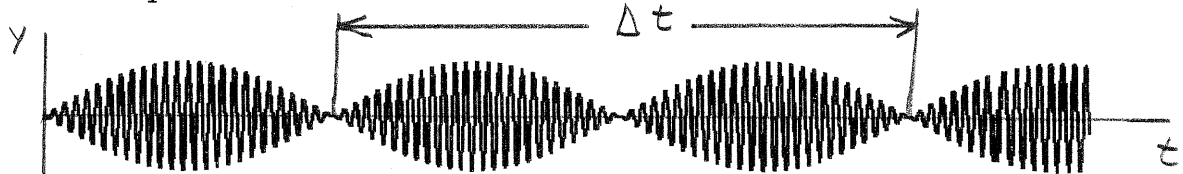
$$p = -B \frac{\partial y}{\partial x}$$

$$= BkA \cos(kx) \sin(\omega t)$$

3. [14 pts] A microphone is placed at rest between two speakers which emit tones of 52 Hz from one speaker to the left of the microphone and 50 Hz from the other speaker to the right of the microphone, as shown in the figure below. The speed of sound is 344 m/s.



The sound signal recorded by the microphone is shown below. Note the beat pattern.



a) (4) Determine the indicated time, Δt , shown in the plot above.

$$f_{\text{beat}} = f_1 - f_2 = 2 \text{ Hz}$$

$$T_{\text{beat}} = \frac{1}{2} \text{ sec} \quad \Delta t = 2 T_{\text{beat}} = 1 \text{ sec}$$

$\Delta t = 1 \text{ sec}$

b) (4) The microphone is now moved at a constant speed in such a way as to eliminate the beats. Should the microphone be moved to the left or to the right? (circle the correct answer) You can ignore path difference interference effects.

<input type="checkbox"/> <- To the left	<input checked="" type="checkbox"/> To the right ->
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c) (6) What is the speed, v_L , of the microphone that is needed to eliminate the beats in the signal recorded by the microphone.

$v_L \equiv$ speed of mic.

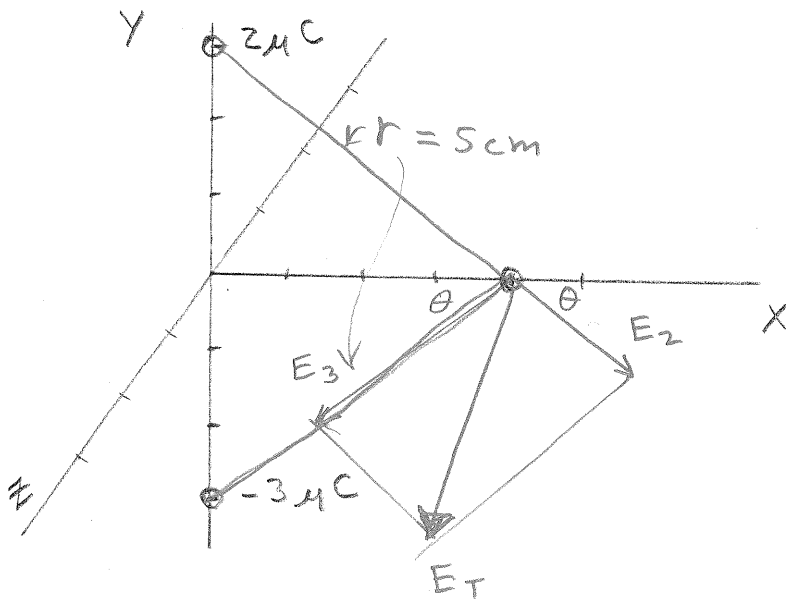
$$f_L = \frac{v + v_L}{v} f_2 = \frac{v - v_L}{v} f_1$$

$$\Rightarrow v f_2 + v_L f_2 = v f_1 - v_L f_1 \Rightarrow v_L (f_2 + f_1) = v (f_1 - f_2)$$

$$\Rightarrow v_L = \left(\frac{f_1 - f_2}{f_1 + f_2} \right) v = \frac{2}{102} 344 \text{ m/s}$$

$v_L = 6.745 \text{ m/s}$

4. [14 pts] A charge of $2 \mu\text{C}$ is placed at $x=0, y=3\text{cm}, z=0$ ($0, 3\text{cm}, 0$). Another charge of $-3 \mu\text{C}$ is placed at $(0, -3\text{cm}, 0)$. Calculate the electric field (E_x, E_y, E_z) at $(4\text{cm}, 0, 0)$.



$$\cos \theta = \frac{4}{5}$$

$$\sin \theta = \frac{3}{5}$$

$$E_x = E_{3x} + E_{2x} = \frac{k(-3\mu\text{C})}{(5\text{cm})^2} \left(\frac{4}{5}\right) + k \frac{2\mu\text{C}}{(5\text{cm})^2} \left(\frac{4}{5}\right)$$

$$= -k \frac{1\mu\text{C}}{(5\text{cm})^2} \frac{4}{5} = -\left(8.988 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(10^{-6}\text{C})}{5^2 (10^{-2}\text{m})^2} \frac{4}{5}$$

$$= -2.8762 \times 10^6 \frac{\text{N}}{\text{C}}$$

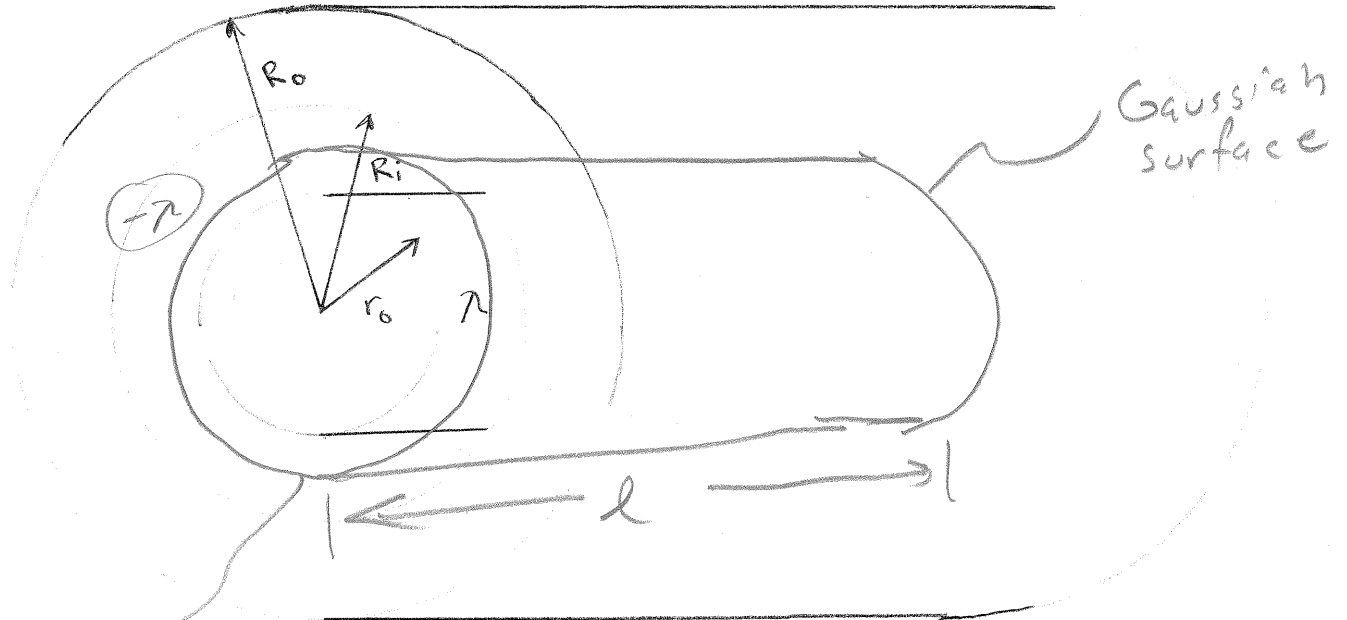
$$E_y = E_{3y} + E_{2y} = -k \frac{3\mu\text{C}}{(5\text{cm})^2} \left(\frac{3}{5}\right) - k \frac{2\mu\text{C}}{(5\text{cm})^2} \left(\frac{3}{5}\right)$$

$$= -k \frac{5\mu\text{C}}{(5\text{cm})^2} \left(\frac{3}{5}\right) = -k \frac{1\mu\text{C}}{(5\text{cm})^2} (3)$$

$$= -3 \left(\frac{5}{4}\right) E_y = -1.079 \times 10^7 \frac{\text{N}}{\text{C}}$$

$E_x = -2.88 \times 10^6 \frac{\text{N}}{\text{C}}$	$\left[\frac{\text{Volts}}{\text{m}} \right]$
$E_y = -1.08 \times 10^7 \frac{\text{N}}{\text{C}}$	
$E_z = 0$	

5. [12 pts] **Gauss's Law:** A hollow conducting cylinder with inner radius R_i and outer radius R_o , has at its center a solid cylindrical conducting wire with outer radius r_o . There is air between the conducting wire and the outer conducting cylindrical shell. There is charge uniformly distributed on the outside of the inner conductor (at $r=r_o$) with charge per unit length λ . There is no charge on the outer surface of the outer conductor. The charge distributions on both conductors are in static equilibrium. Determine the magnitude of the electric field, $E_i(r)$, as a function of radial distance from the axis of the wire, r , in the four regions.



$$\oint \vec{E} \cdot d\vec{A} = E (l 2\pi r) = \frac{\lambda l}{\epsilon_0} \Rightarrow E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$

in conductor

$0 < r < r_o$	$E(r) = 0$
$r_o < r < R_i$	$E(r) = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} \hat{r}$
$R_i < r < R_o$	$E(r) = 0$
$R_o < r$	$E(r) = 0$

in conductor

No net charge inside

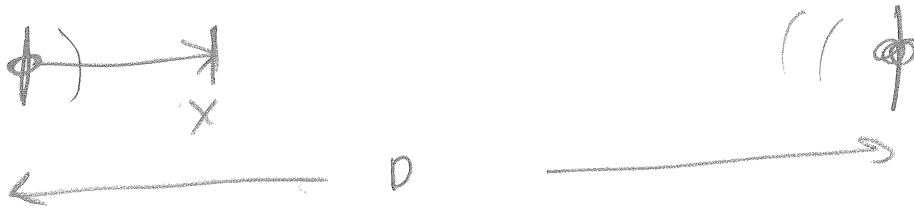
6. [12 pts] A spherical sound wave is emitted by a point source. How many decibels does the sound intensity level change when you move from 1 meter from the source to 10 meters from the source?

$$\begin{aligned}\Delta\beta &= \beta_{10} - \beta_1 = 10 \text{ dB} \left[\log \frac{I_{10}}{I_0} - \log \frac{I_1}{I_0} \right] \\ &= 10 \text{ dB} \log \frac{I_{10}}{I_1} = 10 \text{ dB} \log \left(\frac{r_1^2}{r_{10}^2} \right) \\ &= 10 \text{ dB} \log \left(\frac{1^2}{10^2} \right) = 20 \text{ dB} \log(10^{-1}) = -20 \text{ dB}\end{aligned}$$

$$\Delta\beta = -20 \text{ dB}$$

7. [10 pts] Two Speakers, A, and B, are driven by the same 600Hz signal. The speakers are 3 meters apart. A microphone measuring the sound coming from both speakers is moved from speaker A toward speaker B. What is the closest distance to speaker A that the microphone will measure the first interference minima, X_{\min} (due to path length difference). The speed of sound is 344 m/s.

$$\lambda = \frac{344 \text{ m/s}}{600 \text{ Hz}}$$



$$d = (D - x) - x = D - 2x = n \frac{\lambda}{2} \quad n \text{ odd}$$

$$\Rightarrow x = \frac{1}{2} \left(D - n \frac{\lambda}{2} \right) \quad \text{for } x=0 \quad n = \frac{2D}{\lambda} = \frac{2(3\text{m})}{344(\text{m/s})} = 10.46$$

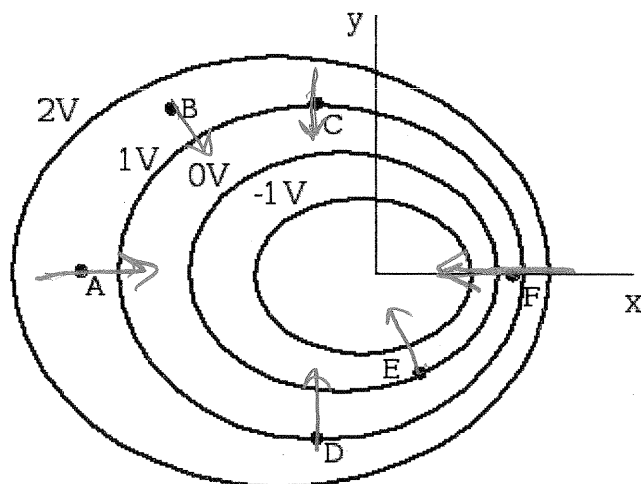
$$\Rightarrow n = 9$$

$$\Rightarrow x = \frac{1}{2} \left[3\text{m} - \frac{9}{2} \left(\frac{344}{600} \right) \right]$$

$$\frac{1}{2} [3\text{m} - 2.58\text{m}]$$

$$X_{\min} = 0.21 \text{ m}$$

8. [12 pts] Below is shown equipotential lines (like in your lab) with the corresponding potentials labeled. The corresponding electric field lies in the plane of the paper (it's 2D, like in the lab). You do not have to show work in this problem.



a) (3) Of all the labeled points, which labeled point has the largest electric field?
(circle one)

A B C D E F

b) (3) Of all the labeled points, which labeled point has the smallest electric field?
(circle one)

A B C D E F

c) (3) Of all the labeled points, which labeled point has the electric field in the x-direction?
(circle one)

A B C D E F

c) (3) Of all the labeled points, which labeled point has the electric field in the negative y-direction?
(circle one)

A B C D E F