Physics 2306 Exam 2 Summer 1, 2005

Name Key **Student ID** ______________

Check your (CRN) section number:

☐ **61082 8:00AM - 9:15AM**

☐ **61084 9:30AM - 10:45AM**

To get credit you must show work.

Problem	*Possible Points*	*Points Scored*
1	**8**	
2	**8**	
3	**8**	
4	**10**	
5	**10**	
6	**10**	
7	**6**	
8	**8**	
9	**8**	
10	**10**	
11	**7**	
12	**7**	
Total	**100**	

Scaled	

1. [8 pts] A parallel plate capacitor (with no dielectric) is charged to $V_1=12$ volts and has a charge of 3 nC (3×10^{-9} C) on it.

   a) (2) What is the capacitance, C, of this capacitor?

$$C = \frac{Q}{V} = \frac{3\times10^{-9}\,C}{12\,V}$$

$\boxed{C = 2.5\times10^{-10}\ F}$

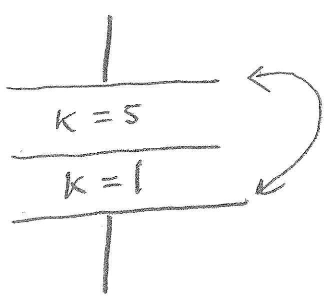
   b) (3) With no source attached the plates of the capacitor are changed to twice their original separation. In this process no charge is transfered to or from the capacitor. What is the potential, V_2, on the capacitor after this is done?

$$Q = C_1V_1 = C_2V_2 = \left[C_1\left(\tfrac{1}{2}\right)\right]V_2$$

$$\Rightarrow V_2 = 2V_1 = 2(12V)$$

$\boxed{V_2 = 24V}$

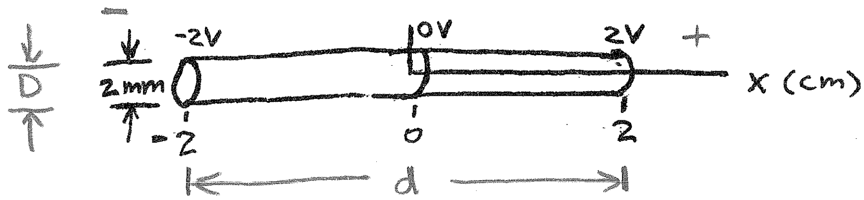
   c) (3) A mica dielectric, with K=5, is inserted between the plates of the capacitor. This dielectric only fills half of the space of the gap of the capacitor and has the same surface area as the parallel plates. What is the potential, V_3, on the capacitor now?



$$V = \frac{V_2}{2} + \frac{V_2}{2K} = \frac{1}{2}\left(1+\frac{1}{K}\right)V_2 = \frac{1}{2}\left(1+\frac{1}{5}\right)24V$$

$\boxed{V_3 = 14.4V}$

2. [8 pts] The figure below shows part of a conductor with resistivity, $\rho = 10^{-5}\ \Omega\text{m}$. Shown is the electric potential at 3 positions in the conductor.



a) (2) What is the direction of the current in the conductor?

direction	$= -\hat{i}$ or $-x$

b) (3) What is the current, I, in the conductor?

$$I = \frac{V}{R} = \frac{V}{\left(\frac{\rho d}{A}\right)} = \frac{VA}{\rho d} = \frac{V\left(\pi \frac{D^2}{4}\right)}{\rho d}$$

$$= \frac{(\cancel{4}V)\ \pi\ (2\times10^{-3}\text{m})^2}{\cancel{4}\ (10^{-5}\,\Omega\,\text{m})\ (.04\text{m})}$$

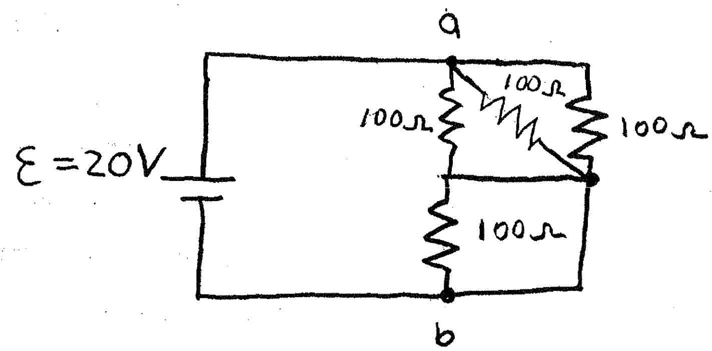
$I = 31.4\ \text{A}$

c) (3) What is the magnitude and direction of the electric field, at x=0, in the conductor?

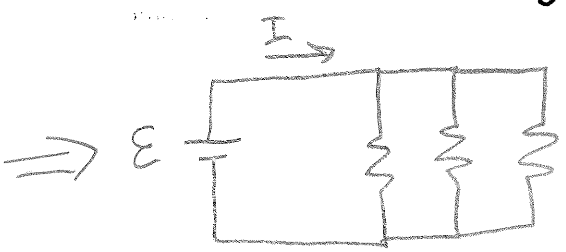
$$E_x = \frac{-\Delta V}{\Delta x} = -\frac{2\text{V}}{.02\text{m}} = -100\ \frac{\text{V}}{\text{m}}$$

$E = 100\ \frac{\text{V}}{\text{m}}$	direction $= -\hat{i}$

3. [8 pts] Solve for the equivalent resistance, R_{eq}, that is between the points labeled a and b, and the current, I, that flows from the ideal battery.



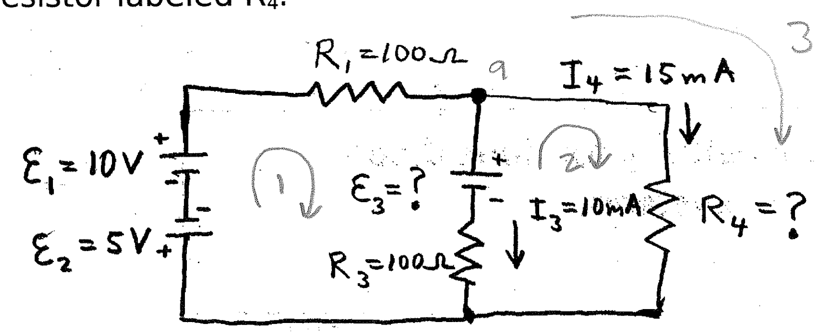
$$I = \frac{V}{R_{eq}} = 3\frac{V}{R} = \frac{3(20)}{100}$$



$\Rightarrow$ $R_{eq} = \frac{R}{3}$

R_{eq} =	$33.\bar{3}\ \Omega$
I =	0.6 A

4. [10 pts] All the EMF (voltage) sources are ideal in the circuit shown. Find the current through the resistor R_1, I_1, the EMF ε_3, and the resistance of the resistor labeled R_4.



$$I_1 = I_3 + I_4 = 10\,mA + 15\,mA = 25\,mA$$

$$\text{KCL (a)} \Rightarrow I_1 - I_4 - I_3 = 0 \Rightarrow$$

$$\text{KVL (1)} \Rightarrow 5V - \underbrace{R_1 I_1}_{2.5V} - \varepsilon_3 - \underbrace{I_3 R_3}_{1.0V} = 0 \Rightarrow \varepsilon_3 = 5V - 3.5V$$

$$\text{KVL (2)} = \varepsilon_3 - I_4 R_4 + I_3 R_3 = 0$$

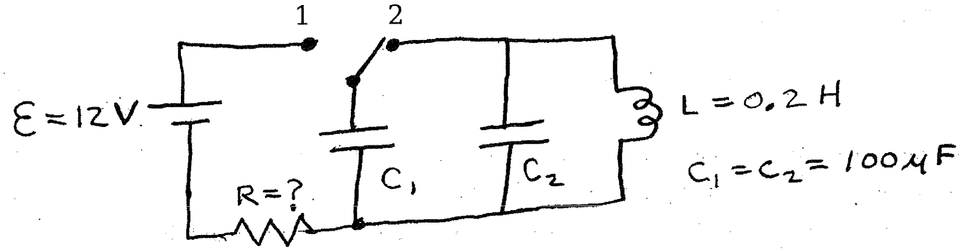
$$\Rightarrow I_4 R_4 = \varepsilon_3 + I_3 R_3 \Rightarrow R_4 = \frac{\varepsilon_3 + I_3 R_3}{I_4}$$

$$= \frac{1.5V + 1.0V}{15\,mA} =$$

I_1 =	25 mA
ε_3 =	1.5 V
R_4 =	$166.\bar{6}\ \Omega$

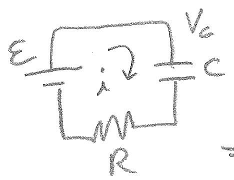
KVL (3) can be used too.

5. [10 pts] The circuit shown has no changes or currents to start with. All circuit components are ideal.



$C_{eq} = 200\mu F$

a) (2 pts) The switch is switched to position 1. The capacitor, C_1, takes 8 seconds to charge to 10 volts. What is R?



$$V_c(t) = V_{max}\left(1 - e^{-t/RC}\right) \Rightarrow \frac{t}{RC} = -\ln\left(1 - \frac{V_c}{V_{max}}\right)$$

$$\Rightarrow \frac{V_c}{V_{max}} = 1 - e^{-t/RC} \Rightarrow \left(1 - \frac{V_c}{V_{max}}\right) = e^{-t/RC}$$

R = 44,648 Ω

b) (2 pts) When the capacitor, C_1, is charged to 10 volts, the switch is moved to the position labeled 2. What will be the current, i_L, just after the switch is moved?

$$\Rightarrow R = \frac{-t}{C \ln\left(1 - \frac{V_c}{V_{max}}\right)} = \frac{-8\,s}{100\times10^{-6}F \ \ln\left(1 - \frac{10}{12}\right)}$$

i_L = 0

c) (3 pts) What will be the maximum current, i_{Lmax}, through the inductor?

Energy

C_2 starts with no Energy

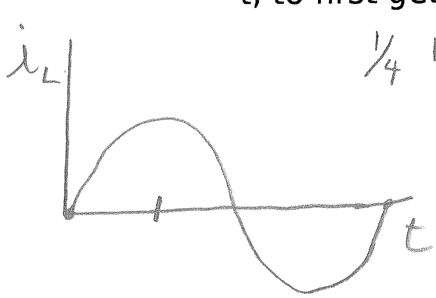
$$E_0 = \frac{1}{2} C_1 V^2 = E_f = \frac{1}{2} L i_{Lmax}^2$$

$$\Rightarrow i_{Lmax}^2 = \frac{C}{L} V^2 \Rightarrow i_{Lmax} = \sqrt{\frac{C_1}{L}}\, V = \sqrt{\frac{100\times10^{-6}\,F}{0.2\,H}}\,(10\,V)$$

$= 0.223606797$ A

i_{Lmax} = 0.224 A

d) (3pts) How long, after the switch is moved to position 2, will it take, t, to first get this current, i_{Lmax}, in part c?



¼ period

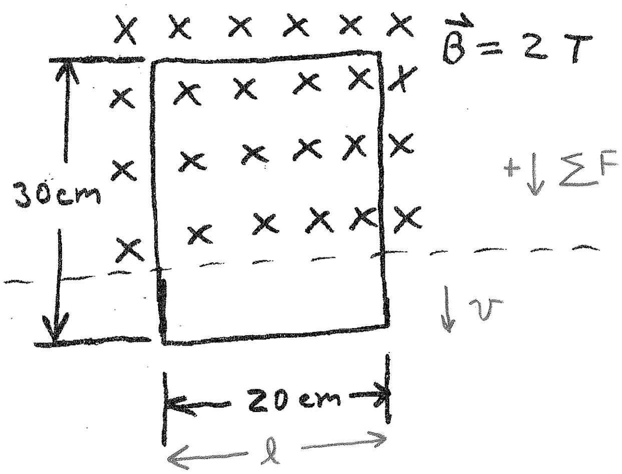
$$= \frac{T}{4} = \frac{1}{4}\frac{(2\pi)}{\omega} = \frac{2\pi}{4}\frac{1}{\sqrt{\frac{1}{LC}}} = \frac{2\pi\sqrt{LC_{eq}}}{4}$$

$$= \frac{\pi}{2}\sqrt{(0.2\,H)\,200\times10^{-6}F}$$

$= 0.00993459$

t = 9.935 ms

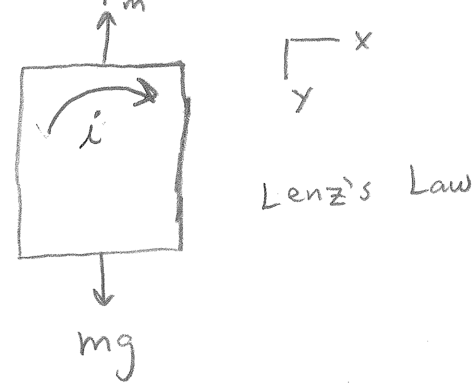
6. [10 pts] The rectangular conducting loop, shown, has a resistance of R=0.01 ohms and a mass of m=10 grams. The loop starts at rest and is released so that the lower-most part of the loop is just outside the uniform magnetic field. When the loop is released two forces act on it, the force of the magnetic field on the current is the top wire (the side wire forces cancel out each other) and the force of gravity (g = 9.8 m/s²). Find a) (2) the direction of the induced current in the loop, b) (3) the initial acceleration, a(0), of the loop when it is first released, and c) (5) the acceleration, a, of the loop for all time while it is still partially in the magnetic field as a function of, **v**, the speed of the loop.



FBD loop

$+\downarrow \sum F = mg - F_m = ma_y$

$F_m = i \vec{l} \times \vec{B}$



$$F_m = l i B \qquad i = \frac{\mathcal{E}}{R} = \frac{\frac{d\Phi_B}{dt}}{R} = \frac{1}{R}\frac{d}{dt}(B l v t) = \frac{B l v}{R}$$

$$\Rightarrow F_m = l\left(\frac{B l v}{R}\right)B = \frac{B^2 l^2 v}{R}$$

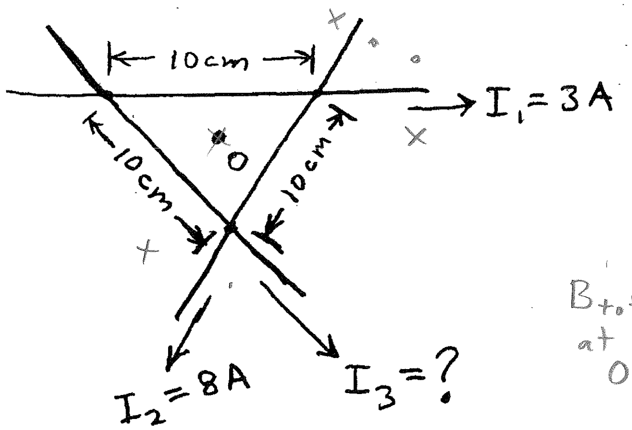
$$\Rightarrow a_y = g - \frac{B^2 l^2}{mR} v = 9.8\,m/s^2 - \frac{(2T)^2(0.2m)^2}{(.01\,kg)(0.01\,\Omega)} v$$

$$= 9.8\,m/s^2 - \left(1600\,\tfrac{1}{s}\right) v \qquad v_t = 0.6125\ cm/s$$

Direction (circle one) = counterclockwise clockwise	
a(0) = $9.8\,m/s^2$	a = $9.8\,m/s^2 - (1600\,1/s)\,v$

$a(0) = g$

7. [6 pts] What must be the current, I_3, so that the magnetic field is zero at the point, O, which is equidistant from the three conductor intersection points?

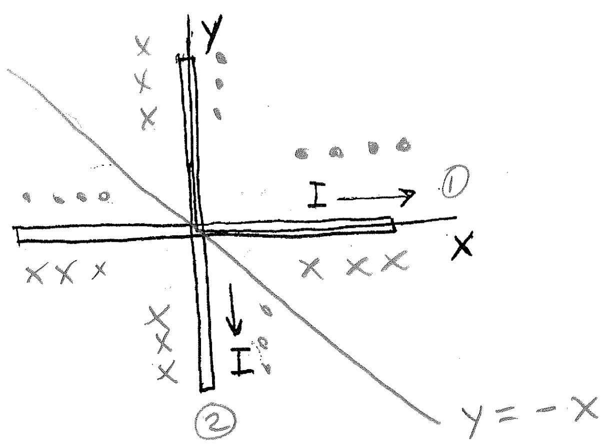


I_1 and I_2 add to the B-field into the paper at O.

$$B_{total\ at\ O} = \frac{\mu_0 I_1}{2\pi r} + \frac{\mu_0 I_2}{2\pi r} - \frac{\mu_0 I_3}{2\pi r}$$
$$= \frac{\mu_0}{2\pi r}(I_1 + I_2 - I_3) = 0$$
$$\Rightarrow I_3 = I_1 + I_2 = 11\ A$$

$I_3 =$ 11 A

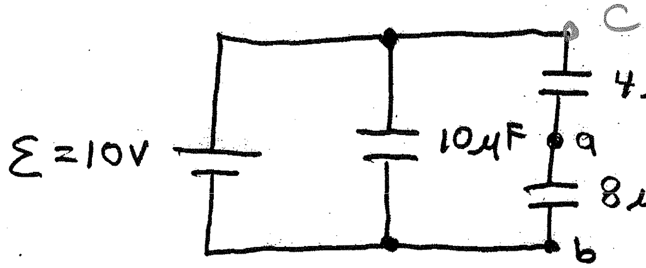
8. [8 pts] The two very long conductors shown both carry, I, 5 amps. Find all positions where the magnetic field from these two currents is zero.



Plotting the fields with × × × and • • • shows the on the line y = -x the two Fields the two fields cancel each other.

At positions: where y = -x

9. [8 pts] Find V_{ab} for the following circuit.



$V_1 = \frac{Q}{C_1}$

$V_2 = \frac{Q}{C_2}$

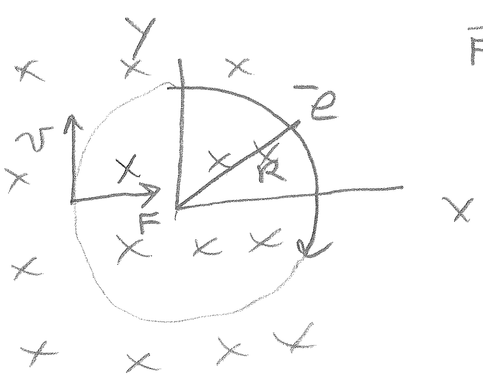
$$C_{eq} = \frac{Q}{V_1 + V_2} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} \qquad Q = V_{cb} C_{eq}$$

$$V_{ab} = V_2 = \frac{Q}{C_2} = \frac{V_{cb}(C_{eq})}{C_2} = \frac{V_{cb}}{\cancel{C_2}} \frac{C_1 \cancel{C_2}}{C_1 + C_2} = \mathcal{E} \frac{C_1}{C_1 + C_2}$$

$$= 10V \left(\frac{4}{4+8} \right)$$

V_{ab} = $3.3\bar{3}$ V

10. [10 pts] An electron, with mass $m_e = 9.11 \times 10^{-31}$ kg, moves in a circle, of radius R=0.5m, in the x-y plane in the clockwise direction, with a speed of v = 6.5 x 10^6 m/s. This motion is caused by the forces from a uniform magnetic field. What are the three vector components of this magnetic field, B_x, B_y, and B_z? The natural cha~~n~~rge constant e = 1.6 x 10^{-19} C.



$$\vec{F} = q\vec{v} \times \vec{B} \qquad |\vec{F}| = evB = m|\vec{a}| = m\frac{v^2}{R}$$

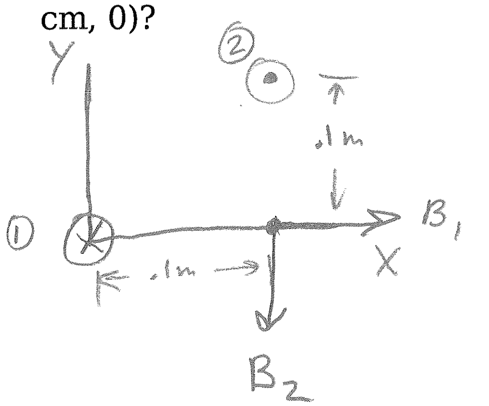
$$\Rightarrow e\cancel{v}B = \frac{mv^{\cancel{2}}}{R}$$

$$\Rightarrow B = \frac{m}{e}\frac{v}{R} = \frac{9.11 \times 10^{-31} kg \; (6.5 \times 10^6 \; m/s)}{(1.6 \times 10^{-19} C)(0.5m)}$$

$$= 7.401875 \times 10^{-5} \; T$$

B_x =	0
B_y =	0
B_z =	-7.40×10^{-5} T

11. [7 pts] A steady current, of 2 amps, flows through a long wire in the z-direction at the x-y position x= 10 cm, y= 10 cm (10 cm, 10 cm). Another steady current, of 2 amps, flows through a long wire in the negative z-direction at the x-y position x= 0 cm, y= 0 cm (0,0). What is the magnetic field, B_x, B_y, B_z, from these two currents at position x= 10 cm, y= 0 cm (10 cm, 0)?

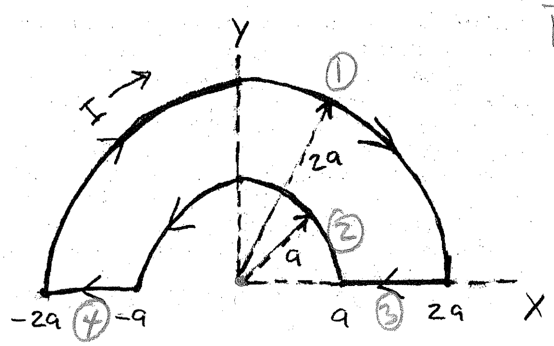


$$B_1 = B_2 = \frac{\mu_0 I}{2\pi r} = \frac{(\cancel{4\pi}^{2} \times 10^{-7}\ Tm/A)(2A)}{\cancel{2\pi}(0.1m)}$$

$$= 4 \times 10^{-6}\ T$$

B_x =	4×10^{-6} T
B_y =	-4×10^{-6} T
B_z =	0

12.[7 pts] The loop shown carries a current of I in the direction shown. Find the magnetic field, **B**, at the origin. Express your answer in terms of I, and a.



$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$$

$$= (-B_1 + B_2)\hat{k} + 0 + 0$$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$B_2 = \int_0^{\pi} \frac{\mu_0}{4\pi} \frac{(\cancel{a}\, d\theta)}{a^{\cancel{2}}} I = \frac{\mu_0 I}{4\pi a}(\pi) = \frac{\mu_0 I}{4a}$$

$$B_1 = B_2(a \to 2a) = \frac{\mu_0 I}{8a}$$

$$\Rightarrow |\vec{B}| = -\frac{\mu_0 I}{8a} + \frac{\mu_0 I}{4a}$$

B_x =	0
B_y =	0
B_z =	$\frac{\mu_0 I}{8a}$