Problems from MasteringPhysics with minor clarifications.

15.3 - Speed, Wavelength, Frequency

The speed of sound in air at 20°C is v =344m/s.

Part A

What is the wavelength, λ, of a sound wave with a frequency of f = 784 Hz, corresponding to the note G_5 on a piano?

$$v = \lambda f \quad \Rightarrow \quad \lambda = \frac{v}{f} = \frac{344\ \frac{\text{m}}{\text{s}}}{784\ \text{Hz}} = \frac{344\ \frac{\text{m}}{\text{s}}}{784\ \frac{1}{\text{s}}} \approx \boxed{0.439\,\text{m}}$$

Part B

What is the frequency of a sound wave with a wavelength of $\lambda = 6.30 \times 10^{-2}$mm? (This frequency is too high for you to hear.)

$$v = \lambda f \quad \Rightarrow \quad f = \frac{v}{\lambda} = \frac{344\ \frac{\text{m}}{\text{s}}}{6.30 \times 10^{-2}\ \text{mm}}$$

$$= \frac{344\ \frac{\text{m}}{\text{s}}}{6.30 \times 10^{-5}\ \text{m}} \approx 5.46 \times 10^6 \frac{1}{\text{s}} = \boxed{5.46 \times 10^6 \text{Hz}}$$

15.5 - Audible Wavelengths

Provided that the amplitude is sufficiently great, the human ear can respond to longitudinal waves over a range of frequencies from about 20.0 Hz to about 20,000Hz.

Part A

Compute the wavelength corresponding to f =20Hz for waves in air ($v = 344\,\frac{\text{m}}{\text{s}}$).

$$v = \lambda f \quad \Rightarrow \quad \lambda = \frac{v}{f} = \frac{344\ \frac{\text{m}}{\text{s}}}{20\ \text{Hz}} = \frac{344\ \frac{\text{m}}{\text{s}}}{20\ \frac{1}{\text{s}}} = \boxed{17.2\,\text{m}}$$

Part B

Compute the wavelength corresponding to f = 20,000 Hz for waves in air ($v = 344\,\frac{\text{m}}{\text{s}}$).

$$v = \lambda f \quad \Rightarrow \quad \lambda = \frac{v}{f} = \frac{344\ \frac{\text{m}}{\text{s}}}{20,000\ \text{Hz}} = \frac{344\ \frac{\text{m}}{\text{s}}}{20,000\ \frac{1}{\text{s}}}$$

$$= \boxed{1.72 \times 10^{-2}\ \text{m}} = 17.2\text{mm}$$

Part C

Compute the wavelength corresponding to 20 Hz for waves in water ($v = 1480\,\frac{\text{m}}{\text{s}}$).

$$v = \lambda f \quad \Rightarrow \quad \lambda = \frac{v}{f} = \frac{1480\ \frac{\text{m}}{\text{s}}}{20\ \text{Hz}} = \frac{1480\ \frac{\text{m}}{\text{s}}}{20\ \frac{1}{\text{s}}} = \boxed{74\,\text{m}}$$

Note: In MasteringPhysics they say the correct answer is 74.0 m.

Part D

Compute the wavelength corresponding to 20,000 Hz for waves in water ($v = 1480\,\frac{\text{m}}{\text{s}}$).

$$v = \lambda f \quad \Rightarrow \quad \lambda = \frac{v}{f} = \frac{1480\ \frac{\text{m}}{\text{s}}}{20,000\ \text{Hz}} = \frac{1480\ \frac{\text{m}}{\text{s}}}{20,000\ \frac{1}{\text{s}}}$$

$$= \boxed{7.4 \times 10^{-2}\text{m}} = 74\text{mm}$$

Note: In MasteringPhysics they say the correct answer is 7.40×10^{-2}m.

15.6 - Transverse Wave

A certain transverse wave is described by

$$y(x,t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right],$$

where $A = 6.10$ mm, $\lambda = 28.0$ cm, and $T = 3.10 \times 10^{-2}$ s.

Part A

Determine the wave's amplitude.
Enter your answer in meters.

The factor that multiplies the cosine function (cos) is the amplitude. By inspection

$$A = 6.10\,\text{mm} = 6.1\,\text{mm} \times \frac{1\,\text{m}}{1000\,\text{mm}} = \boxed{6.1 \times 10^{-3}\,\text{m}}$$

Part B

Determine the wave's wavelength, λ.
Enter your answer in meters.

By inspection

$$\lambda = 28.0\,\text{cm} = 28.0\,\text{cm} \times \frac{1\,\text{m}}{100\,\text{cm}} = \boxed{0.28\,\text{m}}$$

Part C

Determine the wave's frequency, f.
Enter your answer in hertz (Hz).

$$f = \frac{1}{T} = \frac{1}{3.10 \times 10^{-2}\,\mathrm{s}} \approx 32.258\mathrm{Hz} \approx \boxed{32.3\,\mathrm{Hz}}$$

Part D

Determine the wave's speed of propagation, v.
Enter your answer in meters per second ($\frac{\mathrm{m}}{\mathrm{s}}$).

$$v = f\,\lambda = \frac{\lambda}{T} = \frac{28\,\mathrm{cm}}{3.1 \times 10^{-2}\,\mathrm{s}} = \frac{28\,\mathrm{cm}}{3.1 \times 10^{-2}\,\mathrm{s}} \times \frac{\mathrm{m}}{100\,\mathrm{cm}}$$

$$\approx \boxed{9.03\,\frac{\mathrm{m}}{\mathrm{s}}}$$

Part E

Determine the wave's direction of propagation.
Enter your answer in hertz (Hz).

The wave function is

$$y(x,t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right],$$

where A, λ, and T are positive constants. This may be rewritten as

$$y(x,t) = A\cos\left[\frac{2\pi}{\lambda}\left(x - \frac{\lambda}{T}\,t\right)\right] = A\cos\left[\frac{2\pi}{\lambda}\,(x - v\,t)\right]$$

where $v \equiv \frac{\lambda}{T}$. We see that this has a function form that is a function moving in the positive x-direction, $y = f(x - v\,t)$. So the wave is propagating in the $\boxed{+x \text{ direction}}$.

Of course all this work is not necessary, it's just trying to make the point very clear for this solution.

15.7 - Transverse Waves on a String

Transverse (sinusoidal) waves on a string have wave speed $v = 8.00$m/s, amplitude $A = 0.0700$m, and wavelength $\lambda = 0.320$m. The waves travel in the -x direction, and at $t = 0$ the $x = 0$ end of the string has its maximum upward displacement.

Part A

Find the frequency of these waves.
Express your answer to four significant figures.

$$f = \frac{v}{\lambda} = \frac{8\,\frac{\mathrm{m}}{\mathrm{s}}}{0.32\,\mathrm{m}} = \boxed{25\,\mathrm{Hz}}$$

Part B

Find the period of these waves.
Express your answer to four significant figures.

$$T = \frac{1}{f} = \frac{\lambda}{v} = \frac{0.32\,\mathrm{m}}{8\,\frac{\mathrm{m}}{\mathrm{s}}} = \boxed{0.04\,\mathrm{s}}$$

Part C

Find the wave number of these waves.
Express your answer to four significant figures.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.32\,\mathrm{m}} \approx 19.63495\,\frac{1}{\mathrm{m}} \approx \boxed{19.63\,\frac{\mathrm{rad}}{\mathrm{m}}}$$

Part D

Write a wave function describing the wave.
Express your answer in terms of the variables x and t.
Enter each numeric value to four significant figures.

$$y(x,t) = A\cos\left(k\,x + \omega\,t\right),$$

where A is the amplitude, k is the wave number (Part C) and ω is the angular frequency.

$$\omega = 2\pi f = 2\pi\left(\frac{v}{\lambda}\right) = 2\pi \times \left(\frac{8\,\frac{\mathrm{m}}{\mathrm{s}}}{0.32\,\mathrm{m}}\right)$$

$$\approx 157.07963267\,\frac{1}{\mathrm{s}} \approx 157.1\,\frac{rad}{\mathrm{s}}$$

which gives us

$$y(x,t) \approx \boxed{(0.07\,\mathrm{m})\cos\left(19.63\,\frac{\mathrm{rad}}{\mathrm{m}}\,x + 157.1\,\frac{\mathrm{rad}}{\mathrm{s}}\,t\right)}$$

Not including the rad (radians) unit also gives a valid answer. The wave is traveling to in the negative x direction so the two terms in the cos function are added and have the same sign.

Part E

Find the transverse displacement of a particle at $x = 0.360\text{m}$ at time $t = 0.150\text{s}$.
Express your answer to three significant figures.

$$y(0.360\text{m}, 0.150\text{s})$$
$$\approx (0.07\,\text{m})\cos\left(19.63495\,\frac{1}{\text{m}}\,0.360\text{m} + 157.07963\,\frac{1}{\text{s}}\,0.150\text{s}\right)$$
$$\approx 0.0494974\,\text{m} \approx \boxed{0.0495\,\text{m}}$$

Part F

How much time must elapse from the instant in part (E) until the particle at $x = 0.360\text{m}$ next has maximum upward displacement?
Express your answer to three significant figures.

In general

$$y(x,t) = A\cos(k\,x + \omega t)$$

for any x and t. So y is the maximum, A, when

$$k\,x + \omega t = 2\pi\,N$$

where N is any integer. Solving for t gives

$$t = \frac{2\pi}{\omega}\,N - \frac{k}{\omega}\,x = \frac{\lambda}{v}\,N - \frac{x}{v},$$

where we have used $\omega = 2\pi f = 2\pi\frac{v}{\lambda}$ and $v = \frac{\omega}{k}$. Plugging in $\lambda = 0.320\text{m}$, $v = 8.00\,\frac{\text{m}}{\text{s}}$, and $x = 0.360\text{m}$ we get

$$t = \frac{0.320\text{m}}{8\,\frac{\text{m}}{\text{s}}}\,N - \frac{0.360\,\text{m}}{8\,\frac{\text{m}}{\text{s}}} = (0.04\,N - 0.045)\,\text{s}.$$

We are looking for N that gives us the first non-zero of $\Delta t \equiv t - 0.150\text{s}$.

$$\Delta t = (0.04\,N - 0.045)\,\text{s} - 0.150\text{s} = (0.04\,N - 0.195)\,\text{s}.$$

We see that $\Delta t = 0$ when $N \approx 4.9$, so the first value of Δt that is positive with N an integer is with $N = 5$. So

$$\Delta t = (0.04 \times 5 - 0.195)\,\text{s.} = \boxed{0.005\,\text{s}}.$$

Note: We do not round off any numbers.

15.15 - Speed of Propagation vs. Particle Speed

Part A

The equation

$$y(x,t) = A\cos\left[2\pi f\left(\frac{x}{v} - t\right)\right]$$

may be written as

$$y(x,t) = A\cos\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

$$v_y = \frac{\partial}{\partial t}y(x,t) = \frac{\partial}{\partial t}\left\{A\cos\left[\frac{2\pi}{\lambda}(x - vt)\right]\right\}.$$

Using the chain rule we get

$$v_y = A\left\{\frac{\partial}{\partial\left[\frac{2\pi}{\lambda}(x - vt)\right]}\cos\left[\frac{2\pi}{\lambda}(x - vt)\right]\right\}\left\{\frac{\partial}{\partial t}\left[\frac{2\pi}{\lambda}(x - vt)\right]\right\}$$
$$= A\left\{-\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]\right\}\left\{\frac{\partial}{\partial t}\left[\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t\right]\right\}$$
$$= A\left\{-\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]\right\}\left\{-\frac{2\pi}{\lambda}v\right\}$$
$$\Rightarrow \quad \boxed{v_y = \frac{2\pi\,v}{\lambda}A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]}$$

Part B

Find the maximum speed of a particle of the string.

When v_y is a maximum the sin in the previous answer will be 1. So

$$v_{y\,\text{max}} = \boxed{\frac{2\pi\,v}{\lambda}A}$$

15.17 - Transverse Pulse on a Rubber Tube

One end of a rubber tube of length L, with total mass m_1, is fastened to a fixed support. A cord attached to the other end passes over a pulley and supports an object with a mass of m_2. The tube is struck a transverse blow at one end.

Part A

Find the time, t, required for the pulse to reach the other end.
Take free fall acceleration to be g.

The hanging mass, m_2, will make the tension in the rubber tube, $F = m_2\,g$. The linear mass density of the string is $\mu = \frac{m_1}{L}$. So the speed of a wave on the rubber tube (like a string) is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{m_2\,g}{\frac{m_1}{L}}} = \sqrt{\frac{m_2\,g\,L}{m_1}}.$$

The pulse travels a distance of L in the time, t, that we are looking for, so

$$v\,t = L \quad \Rightarrow \quad t = \frac{L}{v} = \frac{L}{\sqrt{\frac{m_2\,g\,L}{m_1}}} = \boxed{\sqrt{\frac{m_1\,L}{m_2\,g}}}.$$