Problems 15.45,52,63,73 from MasteringPhysics with minor clarifications.

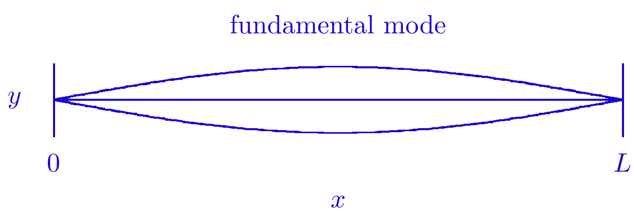
15.45 - Transverse Standing Waves on a Rope

A rope of length L=1.47 m is stretched between two supports with a tension that makes the transverse waves have a speed of v=47.4 m/s.

Part A

What is the wavelength, λ_1, of the fundamental (1st) harmonic?

For a rope (string) with two fixed ends, the 1st harmonic has the longest wavelength (lowest frequency) standing sinusoidal wave that will fit between the two fixed points. So the rope will oscillate like the figure below.



From this figure we see

$$\lambda_1 = 2\,L = 2\,(1.47\,\mathrm{m}) = \boxed{2.94\,\mathrm{m}}$$

Part B

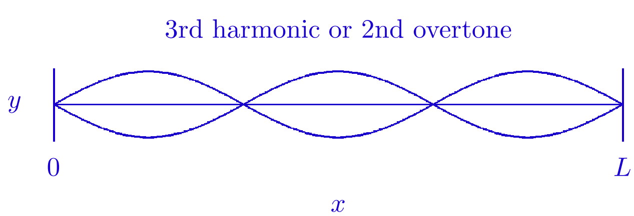
What is the frequency, f_1, of the fundamental harmonic?

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2\,L} = \frac{47.4\,\frac{\mathrm{m}}{\mathrm{s}}}{2\,(1.47\,\mathrm{m})} \approx 16.1224\,\frac{1}{\mathrm{s}} \approx \boxed{16.1\,\mathrm{Hz}}$$

Part C

What is the wavelength, λ_3, of the second overtone?

The second overtone is the 3nd harmonic. It has the third longest wavelength (lowest frequency) standing sinusoidal wave that will fit between the two fixed points. So the rope will oscillate like the figure shown.



From this figure we see

$$\lambda_3 = \frac{2}{3}L = \frac{2}{3}\,(1.47\,\mathrm{m}) = \boxed{0.98\,\mathrm{m}}$$

Part D

What is the frequency, f_3 of the second overtone?

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{2}{3}L} = \frac{47.4\,\frac{\mathrm{m}}{\mathrm{s}}}{\frac{2}{3}\,(1.47\,\mathrm{m})} \approx 48.3673\,\frac{1}{\mathrm{s}} \approx \boxed{48.4\,\mathrm{Hz}}$$

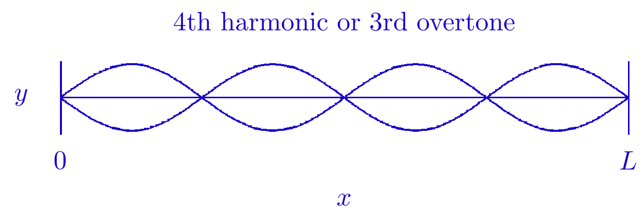
Note:

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{2}{3}L} = 3\,\frac{v}{2\,L} = 3\,f_1$$

Part E

What is the wavelength, λ_4, of the forth harmonic?

It has the forth longest wavelength (lowest frequency) standing sinusoidal wave that will fit between the two fixed points. So the rope will oscillate like the figure shown.



From this figure we see

$$\lambda_4 = \frac{1}{2}L = \frac{1}{2}\,(1.47\,\mathrm{m}) = \boxed{0.735\,\mathrm{m}}$$

Part F

What is the frequency, f_4 of the forth harmonic?

$$f_4 = \frac{v}{\lambda_4} = \frac{v}{\frac{1}{2}L} = 2\,\frac{v}{L} = 4\,\frac{v}{2\,L} = 4f_1 \approx 4 \times 16.1224\,\frac{1}{\mathrm{s}}$$

$$\approx 64.4896\,\mathrm{Hz} \approx \boxed{64.5\,\mathrm{Hz}}$$

15.52 - Weightless Ant

An ant with mass m is standing peacefully on top of a horizontal, stretched rope. The rope has mass per unit length μ and is under tension F. Without warning, Throckmorton starts a sinusoidal transverse wave of wavelength λ propagating along the rope. The motion of the rope is in a vertical plane.

Part A

What minimum wave amplitude, A, will make the ant become momentarily weightless? Assume that m is so small that the presence of the ant has no effect on the propagation of the wave.

The ant will feel weightless when the string exerts no normal force on the ant. This will just happen when the string under the ant is moving with a downward acceleration of g, the acceleration due to gravity. So the amplitude of the vertical acceleration of the string must be g for the ant to feel wieightless, and the direction will be down $(-y)$. Most of the time the acceleration of the string at the ant will be less than g. The vertical acceleration of the string in general is

$$a_y = \frac{\partial^2}{\partial t^2}\left[A \sin(kx - \omega t)\right] = \frac{\partial}{\partial t}\left[-\omega A \cos(kx - \omega t)\right]$$
$$= -\omega^2 A \sin(kx - \omega t)$$

So

$$a_{y\,\mathrm{max}} = \omega^2 A.$$

The ant will feel weightless at one instant in the period of the wave if $a_{y\,\mathrm{max}}$ is g, or

$$g = \omega^2 A \quad \Rightarrow \quad A = \frac{g}{\omega^2} = \frac{g}{(k\,v)^2} = \frac{g}{k^2\,v^2}$$

$$\Rightarrow \quad A = \frac{g}{\left(\frac{2\pi}{\lambda}\right)^2 \frac{F}{\mu}} = \boxed{\frac{g\,\mu\,\lambda^2}{4\,\pi^2\,F}}$$

15.63 - Sinusoidal Transverse Wave

A sinusoidal transverse wave travels on a string. The string has length L and mass m. The wave speed is v and the wavelength is λ.

Part A

If the wave is to have an average power of P_{av}, what must be the amplitude of the wave?

The power in a wave is

$$P(x,t) = F_y v_y = -F \frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t},$$

where $y(x,t)$ is the wave function or transverse displacement of the string as a function of position, x, and time, t. The average power for (averaged over time) for a sinusoidal wave can be written as

$$P_{\mathrm{av}} = \frac{1}{2} F \omega k A^2.$$

Using

$$v^2 = \frac{F}{\mu} = \frac{F}{\frac{m}{L}} = \frac{F\,L}{m} \quad \Rightarrow \quad F = \frac{mv^2}{L},$$

$$k = \frac{2\pi}{\lambda}, \quad \text{and} \quad \omega = kv = \frac{2\pi}{\lambda} v$$

we can write

$$P_{\mathrm{av}} = \frac{1}{2}\left(\frac{mv^2}{L}\right)\left(2\pi\frac{v}{\lambda}\right)\frac{2\pi}{\lambda}A^2 = 2\pi^2 \frac{mv^3}{\lambda^2 L}A^2.$$

Which gives

$$A^2 = \frac{P_{\mathrm{av}}\lambda^2 L}{2\pi^2 m v^3} \quad \Rightarrow \quad A = \sqrt{\frac{P_{\mathrm{av}}\lambda^2 L}{2\pi^2 m v^3}} = \boxed{\sqrt{\frac{P\lambda^2 L}{2\pi^2 m v^3}}}$$

Part B

For this same string, if the amplitude and wavelength are the same as in part A, what is the average power for the wave if the tension is increased such that the wave speed is doubled?

From Part A

$$P_{\mathrm{av}} = 2\pi^2 \frac{mv^3}{\lambda^2 L} A^2,$$

so

$$P_{\mathrm{av}} \propto v^3 \quad \Rightarrow \quad \frac{P_{\mathrm{av}\,2}}{P_{\mathrm{av}\,1}} = \frac{v_2^3}{v_1^3}.$$

So if the speed is doubled, $v_2 = 2v_1$, which gives

$$\frac{P_{\mathrm{av}\,2}}{P_{\mathrm{av}\,1}} = \frac{(2)^3}{1^3} \quad \Rightarrow \quad P_{\mathrm{av}\,2} = 8\,P_{\mathrm{av}\,1} = \boxed{8\,P}.$$

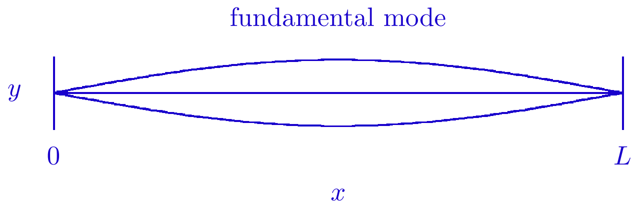
15.73 - Pit and Plank

A wooden plank is placed over a pit that is $L =$5.00m wide. A physics student stands in the middle of the plank and begins to jump up and down such that she jumps upward from the plank two times each second. The plank oscillates with a large amplitude, with maximum amplitude at its center.

Part A

What is the speed of transverse waves, v, on the plank?

It appears that the fundimental mode of the plank is excited. So the plank move like the in the following figure.



The wavelength

$$\lambda = 2L\,.$$

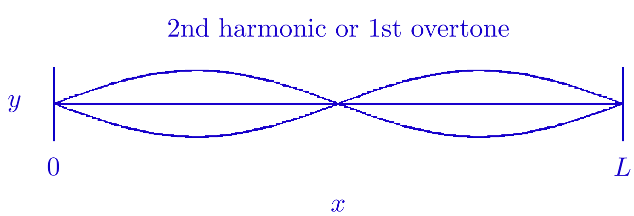
The frequency of the oscillation is $f = \frac{2}{\mathrm{s}}$. So the speed, v, of the wave is

$$v = f\,\lambda = f\,(2L) = \frac{2}{\mathrm{s}} 2\,(5\mathrm{m}) = \boxed{20\,\mathrm{ms}}$$

Part B

At what rate does the student have to jump to produce large-amplitude oscillations if she is standing 1.25m from the edge of the pit? (Note: The transverse standing waves of the plank have nodes at the two ends that rest on the ground on either side of the pit.)

The student would be making a large deflection 1.25m/5m = 1/4 of the way from the end of the plank. The lowest frequency mode that the student can excite this way looks like



So the frequency should be twice the fundimental frequency of 2 Hz.

$$f_2 = 2f_1 = 2\frac{2}{\mathrm{s}} = \boxed{4\,Hz}$$