Problems 16.2,7,10,21,24,26 from MasteringPhysics with minor clarifications.

16.2 - Sound in Water

For sound waves in air with frequency 1000 Hz, a displacement amplitude of 1.2×10^{-8} m produces a pressure amplitude of 3.0×10^{-2} Pa above and below atmospheric pressure $p_a = 1.013 \times 10^5$ Pa. Water at 20°C has a bulk modulus of $B = 2.2 \times 10^9$ Pa, and the speed of sound in water at this temperature is $v = 1480\,\mathrm{m/s}$.

Part A

For 1000Hz sound waves in 20°C water, what displacement amplitude is produced if the pressure amplitude is $p_{\max} = 3.0 \times 10^{-2}$ Pa?

For a sinusoidal wave the differential pressure amplitude, $p_{\max}$, is given by

$$p_{\max} = B\,k\,A\,,$$

where B is the bulk modulus, $k = \frac{2\pi}{\lambda}$ is the wave number, and A is the displacement amplitude. Solving for A we get

$$A = \frac{p_{\max}}{B\,k} = \frac{p_{\max}}{B\,\frac{2\pi}{\lambda}} = \frac{p_{\max}\,\lambda}{2\pi\,B} = \frac{p_{\max}\,\frac{v}{f}}{2\pi\,B} = \frac{p_{\max}\,v}{2\pi\,f\,B}$$

$$= \frac{3.0 \times 10^{-2}\,\mathrm{Pa}\,\left(1480\frac{\mathrm{m}}{\mathrm{s}}\right)}{2\pi\,1000\frac{1}{\mathrm{s}}\,(2.2 \times 10^9\,\mathrm{Pa})} \approx 3.21204 \times 10^{-12}\mathrm{m}$$

$$\approx \boxed{3.21 \times 10^{-12}\mathrm{m}}$$

16.7 - Speed of Sound in Water and in Air

A submerged scuba diver hears the sound of a boat horn directly above her on the surface of the lake. At the same time, a friend on dry land $l_1 = 22.0$ m from the boat also hears the horn. The horn is $l_2 = 2.00$ m above the surface of the water.

Part A

What is the distance from the horn to the diver, L? Both air and water are at 20°C. (The numbers that you had may differ.)

The horn sound must travel an unknow distance L to get to the diver, 2 m of that distance is through air and the rest through water. We can compute the time, t_1, for the sound to travel through the air along land giving

$$t_1 = \frac{l_1}{v_{\mathrm{air}}}\,,$$

where $l_1 = 22$ m is the distance the sound travels, and $v_{\mathrm{air}} = 344$ m/s is the speed of sound in air. The time that the sound spends traveling in the water to get to the diver t_2 is

$$t_2 = t_1 - \frac{l_2}{v_{\mathrm{air}}} = \frac{l_1}{v_{\mathrm{air}}} - \frac{l_2}{v_{\mathrm{air}}} = \frac{l_1 - l_2}{v_{\mathrm{air}}}\,,$$

where $l_2 = 2.00$ m is the distance from the horn to the water toward the diver. The speed of sound in water is $v_{\mathrm{w}} = 1480$ m/s. So from the horn to the diver, L, is

$$L = l_2 + v_{\mathrm{w}}\,t_2 = l_2 + v_{\mathrm{w}}\,\frac{l_1 - l_2}{v_{\mathrm{air}}} = 2\,\mathrm{m} + \frac{1480}{344}\,(22\,\mathrm{m} - 2\,\mathrm{m})$$

$$\approx 88.04651\,\mathrm{m} \approx \boxed{88.0\,\mathrm{m}}$$

16.10 - Speed of Sound verses Temperature

The speed of sound in air at a temperature of T was found to be v.

Part A

What is the change in speed, Δv, for a change in air temperature of ΔT?

In general the speed of sound in air is

$$v = \sqrt{\frac{\gamma\,R\,T}{M}}\,,$$

where γ, R, and M are constants. So

$$v \propto \sqrt{T}\,.$$

So

$$v' = \frac{\sqrt{T'}}{\sqrt{T}}\,v \quad \Rightarrow \quad v' = \frac{\sqrt{T + \Delta T}}{\sqrt{T}}\,v = \sqrt{1 + \frac{\Delta T}{T}}\,v$$

$$\Delta v \equiv v' - v = \sqrt{1 + \frac{\Delta T}{T}}\,v - v = \boxed{v\left(\sqrt{1 + \frac{\Delta T}{T}} - 1\right)}.$$

16.21 - Loud Baby

A baby's mouth is a distance of L_1 from her father's ear and a distance of L_2 from her mother's ear.

Part A

What is the difference between the sound intensity levels heard by the father and by the mother?

$$\beta_F - \beta_M = 10\,\text{dB}\,\log\left(\frac{I_F}{I_0}\right) - 10\,\text{dB}\,\log\left(\frac{I_M}{I_0}\right)$$

$$= 10\,\text{dB}\left[\log\left(\frac{I_F}{I_0}\right) - \log\left(\frac{I_M}{I_0}\right)\right]$$

$$= 10\,\text{dB}\,\log\left(\frac{I_F}{I_0}\frac{I_0}{I_M}\right) = 10\,\text{dB}\,\log\left(\frac{I_F}{I_M}\right)$$

$$= 10\,\text{dB}\,\log\left(\frac{L_2^2}{L_1^2}\right) = 10\,(2)\,\text{dB}\,\log\left(\frac{L_2}{L_1}\right)$$

$$= \boxed{20\,\text{dB}\,\log\left(\frac{L_2}{L_1}\right)}$$

16.24 - Open Pipe

The fundamental frequency of an open pipe is $f_1 = 594$ Hz.

Part A

What is the fundamental frequency, f_1', if one end is plugged?

The length of the pipe, L, and the speed of sound, v, stay constant. For an open pipe length of the pipe, L, is related to the wavelength λ_1 like

$$L = \frac{1}{2}\lambda_1 = \frac{1}{2}\,\frac{v}{f_1}\,.$$

The fundamental frequency if one end is plugged, f_1', is

$$f_1' = \frac{v}{4\,L} = \frac{v}{4\left(\frac{1}{2}\,\frac{v}{f_1'}\right)} = \frac{f_1}{2} = \frac{594\,\text{Hz}}{2} = \boxed{297\,\text{Hz}}$$

16.26 - Open Pipe

Consider a pipe $L = 45.0$ cm long if the pipe is open at both ends. Use $v = 344$ m/s.

Part A

Find the fundamental frequency, f_1.

$$f_1 = \frac{v}{2\,L} = \frac{344\,\frac{\text{m}}{\text{s}}}{2 \times 0.45\text{m}} \approx 382.222\,\text{Hz} \approx \boxed{382\,\text{Hz}}$$

Part B

Find the frequency of the first overtone, f_2.

$$f_2 = \frac{v}{L} = 2\,f_1 = \frac{344\,\frac{\text{m}}{\text{s}}}{0.45\text{m}} \approx 764.4444\,\text{Hz} \approx \boxed{764\,\text{Hz}}$$

Part C

Find the frequency of the second overtone, f_3.

$$f_3 = 3\,f_1 \approx 3 \times 382.222\,\text{Hz} \approx 1146.6666\,\text{Hz} \approx \boxed{1150\,\text{Hz}}$$

Part D

Find the frequency of the third overtone, f_4.

$$f_4 = 4\,f_1 \approx 4 \times 382.222\,\text{Hz} \approx 1528.889\,\text{Hz} \approx \boxed{1530\,\text{Hz}}$$

Part E

What is the number of the highest harmonic that may be heard by a person who can hear frequencies from 20 Hz to $f_{\text{max}} = 20{,}000$ Hz?

Let the highest audible frequency be, f_n which will be less than or equal to f_{max}.

$$f_n = n\,f_1 \le f_{\text{max}} \quad \Rightarrow \quad n \le \frac{f_{\text{max}}}{f_1} \approx \frac{20,000\,\text{Hz}}{382.222\,\text{Hz}} \approx 52.326$$

So the largest n can be is $\boxed{52}$. So

$$f_n = 52\,f_1 \approx 52 \times 382.222\,\text{Hz} \approx 19875.555\,\text{Hz} \approx 19,900\,\text{Hz}$$

Part F

Now pipe is closed at one end. Find the fundamental frequency, f_1.

$$f_1 = \frac{v}{4\,L} = \frac{344\,\frac{\text{m}}{\text{s}}}{4\,0.45\text{m}} \approx 191.11111\,\text{Hz} \approx \boxed{191\,\text{Hz}}$$

Part G

Find the frequency of the first overtone, f_3.

$$f_3 = 3\,f_1 \approx 3 \times 191.11111\,\text{Hz} \approx 573.333\,\text{Hz} \approx \boxed{573\,\text{Hz}}$$

Part H

Find the frequency of the second overtone, f_5.

$$f_5 = 5\,f_1 \approx 5 \times 191.11111\,\text{Hz} \approx 955.5556\,\text{Hz} \approx \boxed{956\,\text{Hz}}$$

Part I

Find the frequency of the third overtone, f_7.

$$f_7 = 7\,f_1 \approx 7\times 191.11111\,\text{Hz} \approx 1337.77778\,\text{Hz} \approx \boxed{1340\,\text{Hz}}$$

Part J

What is the number of the highest harmonic that may be heard by a person who can hear frequencies from 20 Hz to $f_{\text{max}} = 20{,}000$ Hz?

Let the highest audible frequency be, f_n which will be less than or equal to f_{max}.

$$f_n = n\,f_1 \le f_{\text{max}} \quad \Rightarrow \quad n \le \frac{f_{\text{max}}}{f_1} \approx \frac{20,000\,\text{Hz}}{191.11111\,\text{Hz}} \approx 104.65$$

So the largest n can be is $\boxed{103}$, since n is odd. So

$$f_n = 103\,f_1 \approx 103\times 191.11111\,\text{Hz} \approx 19684.4444\,\text{Hz} \approx 19,700\,\text{Hz}$$