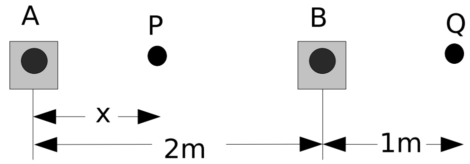
Problems 16.31,52,60,64 from MasteringPhysics with minor clarifications.

16.31 - Sound Interference



Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. Consider point Q along the extension of the line connecting the speakers, 1.00 m to the right of speaker B. Both speakers emit sound waves that travel directly from the speaker to point Q.

Part A

What is the lowest frequency, f_c, for which constructive interference occurs at point Q?

The path length difference, d, is

$$d = (3 - 1)\text{m} = 2\,\text{m}\,.$$

The lowest frequency will will have the longest wavelength. For constructive interference the path length difference, d, should be an integer number of wavelengths. So longest wavelength, λ_c, that has $d = n\lambda_c$, is when $n = 1$, and $d = \lambda_c$ so

$$f_c = \frac{v}{\lambda_c} = \frac{v}{d} = \frac{344\frac{\text{m}}{\text{s}}}{2\text{m}} = \boxed{172\text{Hz}}$$

Part B

What is the lowest frequency, f_d, for which destructive interference occurs at point Q?

The lowest frequency will will have the longest wavelength. For destructive interference the path length difference, d, should be an odd integer number of half wavelengths. So longest wavelength, λ_d, that has $d = n\frac{\lambda_d}{2}$, where n is odd. is when $n = 1$, and $2\,d = \lambda_d$ so

$$f_d = \frac{v}{\lambda_d} = \frac{v}{2\,d} = \frac{344\frac{\text{m}}{\text{s}}}{2 \times 2\text{m}} = \boxed{86.0\text{Hz}}$$

16.52 - A New Musical Instrument

You have designed a new musical instrument of very simple construction. Your design consists of a metal tube with length L and diameter $L/10$. You have stretched a string of mass per unit length μ across the open end of the tube. The other end of the tube is closed. To produce the musical effect you're looking for, you want the frequency of the third-harmonic standing wave on the string to be the same as the fundamental frequency for sound waves in the air column in the tube. The speed of sound waves in this air column is v_s.

Part A

$$f_{3\,\text{string}} = f_{1\,\text{tube}} \quad\Rightarrow\quad 3\,\frac{v_{\text{string}}}{2\,\frac{L}{10}} = \frac{v_s}{4\,L} \quad\Rightarrow\quad \frac{\sqrt{\frac{F}{\mu}}}{\frac{L}{15}} = \frac{v_s}{4\,L}$$

$$\Rightarrow\quad \sqrt{F} = \sqrt{\mu}\frac{L}{15}\frac{v_s}{4\,L} \quad\Rightarrow\quad \boxed{F = \frac{\mu\,v_s^2}{3600}}$$

16.60 - Organ Pipe

The frequency of the note F_4 is f_F.

Part A

If an organ pipe is open at one end and closed at the other, what length, L, must it have for its fundamental mode to produce this note at a temperature of T?
Take the speed of sound to be v_s.

$$f_F = \frac{v_s}{4\,L} \quad\Rightarrow\quad L = \boxed{\frac{v_s}{4\,f_F}}$$

Part B

At what air temperature will the frequency be f? (Ignore the change in length of the pipe due to the temperature change.)

Let the new temperature be T', the original temperature is T, the new speed of sound be v_s', and the length of the tube is L. We need to find T' in terms of the givens f, f_F and T (and v_s is a given too).

$$f = \frac{v_s'}{4\,L} = \frac{v_s\sqrt{\frac{T'}{T}}}{4\,L} \quad\Rightarrow\quad \sqrt{\frac{T'}{T}} = \frac{4\,L\,f}{v_s} = \frac{f}{f_F}$$

$$\Rightarrow\quad T' = \boxed{T\left(\frac{f}{f_F}\right)^2}$$

16.64 - Bat, Doppler, and Beats

A bat flies toward a wall, emitting a steady sound of frequency $f_s = 2.00$ kHz. This bat hears its own sound plus the sound reflected by the wall.

Part A

How fast should the bat fly in order to hear a beat frequency of $f_{\text{beat}} = 10.0$ Hz? Give your answer to two significant figures.

Take the speed of sound to be v =344 m/s.

We'll use the Doppler effect equation (16.29 from text)

$$f_L = \frac{v + v_L}{v + v_S} f_S \,,$$

where f_L is the frequency of the sound received by the listener, v is the speed of sound, v_L is the speed of the listener moving toward the source, v_S is the speed of the source moving away from the listener, and f_S is the frequency of the sound source.

We'll use it twice, once when the sound goes to the wall and once when the sound goes back to the bat.

Let v_B be the speed of the bat toward the wall, the f_W be the frequency of the sound that is reflected from the wall, and f_B be the frequency received by the bat from the reflection. For calculating F_W, the wall is not moving $v_L = 0$, and the source (bat) is moving toward the listener (wall) $v_S = -v_B$, so

$$f_W = \frac{v}{v - v_B} \, f_s \,.$$

Next the source and listener are switched when the sound goes back to the bat from the wall; we have $v_S = 0$ and $v_L = v_B$ giving

$$f_B = \frac{v + v_B}{v} \, f_W = \frac{v + v_B}{v} \left(\frac{v}{v - v_B} \, f_s \right) = \frac{v + v_B}{v - v_B} \, f_s \,.$$

So

$$f_{\text{beat}} = f_B - f_s = \frac{v + v_B}{v - v_B} \, f_s - f_s$$

$$= \frac{v + v_B - (v - v_B)}{v - v_B} \, f_s = \frac{2 \, v_B}{v - v_B} \, f_s \,.$$

So

$$(v - v_B) \, f_{\text{beat}} = 2 \, v_B \, f_s \quad \Rightarrow \quad v \, f_{\text{beat}} - v_B \, f_{\text{beat}} = 2 \, v_B \, f_s$$

$$\Rightarrow \quad v_B \, (f_{\text{beat}} + 2 \, f_s) = v \, f_{\text{beat}} \quad \Rightarrow \quad v_B = v \, \frac{f_{\text{beat}}}{f_{\text{beat}} + 2 \, f_s}$$

$$= 344 \, \frac{\text{m}}{\text{s}} \left(\frac{10}{10 + 2 \, (2{,}000)} \right) \approx 0.8578 \, \frac{\text{m}}{\text{s}} \approx \boxed{0.86 \, \frac{\text{m}}{\text{s}}}$$
