Problems 21.8,20,22 from MasteringPhysics with minor clarifications.

21.8 - Charged Aluminum Spheres

Two small aluminum spheres, each having mass $m = 0.0250$ kg, are separated by $l = 80.0$ cm.

Part A

How many electrons, N, does each sphere contain? (The atomic mass of aluminum is $M = 26.982$ g/mol, and its atomic number is 13.)

$$N = 13\,\frac{m}{M}N_a = 13\,\frac{25\text{g}}{26.982\text{g/mol}}6.022 \times 10^{23}\frac{1}{\text{mol}}$$

$$\approx 7.25354 \times 10^{24} \approx \boxed{7.25 \times 10^{24}}$$

Part B

How many electrons would have to be removed from one sphere and added to the other to cause an attractive force between the spheres of magnitude $F = 1.00 \times 10^4$ N (roughly one ton)? Assume that the spheres may be treated as point charges.

The magnitude of the change on both spheres will be the came, but opposite in sign. Using Coulomb's law we have

$$F = k\frac{q^2}{l^2} = k\frac{n^2 e^2}{l^2} \quad \Rightarrow \quad n^2 = \frac{F}{k}\,\frac{l^2}{e^2}$$

$$\Rightarrow \quad n = \sqrt{\frac{F}{k}}\,\frac{l}{e} \approx \sqrt{\frac{10^4\,\text{N}}{8.988 \times 10^9\,\frac{\text{N m}^2}{\text{C}^2}}}\,\frac{0.8\text{m}}{1.6 \times 10^{-19}\text{C}}$$

$$\approx 5.27398 \times 10^{15} \approx \boxed{5.27 \times 10^{15}}$$

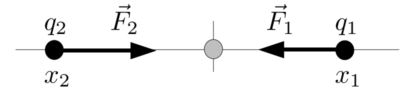
Part C

What fraction of all the electrons in each sphere does this represent?

$$\frac{n}{N} \approx \frac{5.27398 \times 10^{15}}{7.25354 \times 10^{24}} \approx 7.27094 \times 10^{-10} \approx \boxed{7.27 \times 10^{-10}}$$

21.8 - Two Point Charges

Two point charges are placed on the x-axis as follows: one positive charge, q_1, is located to the right of the origin at $x{=}x_1$, and a second positive charge, q_2, is located to the left of the origin at $x{=}x_2$.



Part A

What is the total force (magnitude and direction) exerted by these two charges on a negative point charge, q_3, that is placed at the origin?
Use ϵ_0 for the permittivity of free space. Take positive forces to be along the positive x-axis. Do not use unit vectors.

Keeping in mind that q_3 is less than zero we get

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = k\frac{q_1\,q_3}{x_1^2}\,\hat{i} - k\frac{q_2\,q_3}{x_2^2}\,\hat{i}$$

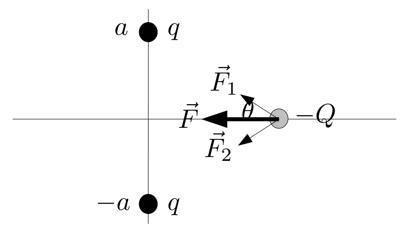
$$= \frac{1}{4\pi\epsilon_0}\,\frac{q_1\,q_3}{x_1^2}\,\hat{i} - \frac{1}{4\pi\epsilon_0}\,\frac{q_2\,q_3}{x_2^2}\,\hat{i} = \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{x_1^2} - \frac{q_2}{x_2^2}\right)\hat{i}\,.$$

The instructor got this wrong. Apparently q_3 is the magnitude of the charge, and removing the unit vector ($\hat{i}$) for the MasteringPhysics answer we get

$$F = \boxed{\frac{-q_3}{4\pi\epsilon_0}\left(\frac{q_1}{x_1^2} - \frac{q_2}{x_2^2}\right)}\,.$$

21.22 - Two Point Charges

Two positive point charges q are placed on the y-axis at $y{=}a$ and $y{=}{-a}$. A negative point charge $-Q$ is located at some point on the $+x$-axis.



Part A

Find the x-component of the net force that the two positive charges exert on $-Q$. (Your answer should only involve k, q, Q, a, and the coordinate x of the third charge.)

From symmetry the magnetude of $\vec{F}_1$, F_1, is the same as magnetude of $\vec{F}_2$, F_2, and so are the x and y components.

$$F_x = F_{1\,x} + F_{2\,x} = 2\,F_{1\,x} = -2\,F_1\,\cos\theta$$

$$= -2\left(k\frac{q\,Q}{x^2+a^2}\right)\frac{x}{\sqrt{x^2+a^2}} = \boxed{\frac{-2\,k\,q\,Q\,x}{(x^2+a^2)^{\frac{3}{2}}}}$$

Part B

Find the y-component of the net force that the two positive charges exert on $-Q$. (Your answer should only involve k, q, Q, a, and the coordinate x of the third charge.)

From symmetry the y-component of the net force from both q charges are equal and opposing, so

$$F_y = \boxed{0}.$$

Part C

What is the net force on the charge $-Q$ when it is at the origin (x=0)?

From symmetry the x-component of the force either q charge is zero, and as in Part B, from symmetry the y-component of the net force from both q charges are equal and opposing, so

$$F = \boxed{0}.$$
