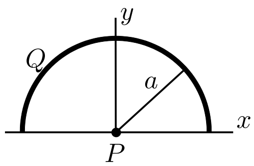
Problem 21.94 from MasteringPhysics with minor clarifications.

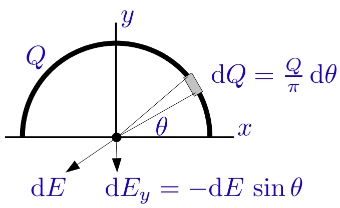
21.94 - Semicircle of Charge



Positive charge Q is uniformly distributed around a semicircle of radius a. The center of curvature, point P, is at the origin.

Part A

Find the magnitude of the electric field at the center of curvature P. (Your answer should only involve k, Q, and a.)



From symmetry the total x-component of the electric field will be zero. So we will compute the y-component for a small piece of the semicircle of length $\mathrm{d}s = a\,\mathrm{d}\theta$ with charge on this small length being

$$\mathrm{d}Q = \frac{Q}{\pi\,a}\,\mathrm{d}s = \frac{Q}{\pi\,a}\,(a\,\mathrm{d}\theta) = \frac{Q}{\pi}\,\mathrm{d}\theta\,.$$

So the magnitude of the electric field from this small piece of charge is

$$\mathrm{d}E = k\frac{\mathrm{d}Q}{a^2} = k\frac{\frac{Q}{\pi}\,\mathrm{d}\theta}{a^2} = k\frac{Q}{\pi a^2}\,\mathrm{d}\theta\,.$$

The y-component of this small electric field is

$$\mathrm{d}E_y = -\mathrm{d}E\,\sin\theta = -\left(k\frac{Q}{\pi a^2}\,\mathrm{d}\theta\right)\sin\theta = -k\frac{Q}{\pi\,a^2}\,\sin\theta\,\mathrm{d}\theta\,.$$

We sum over all values of θ to get the total y-component of the electric field

$$E_y = \int_{\theta=0}^{\pi} -k\frac{Q}{\pi\,a^2}\,\sin\theta\,\mathrm{d}\theta = -k\frac{Q}{\pi\,a^2}\int_{\theta=0}^{\pi}\sin\theta\,\mathrm{d}\theta$$
$$= -k\frac{Q}{\pi\,a^2}\left(-\cos\theta|_{\theta=0}^{\pi}\right) = -k\frac{Q}{\pi\,a^2}\,(1+1) = -\frac{2\,k\,Q}{\pi\,a^2}\,.$$

So the magnitude of E_y is

$$|E_y| = \boxed{\frac{2\,k\,Q}{\pi\,a^2}}\,.$$

Part B

What is the direction of the electric field at the center of curvature P.

From Part A we see that the electric field is in the negative y-direction. So the direction of the electric field is $\boxed{\text{downward}}$.
