Problem 22.1,4,6,32,54 from MasteringPhysics with minor clarifications.

22.1 - Electric Flux through a Flat Sheet

A flat sheet of paper of area $A = 0.105\text{m}^2$ is oriented so that the normal to the sheet is at an angle of $\phi = 61.0°$ to a uniform electric field of magnitude $E = 13.0$ N/C.

Part A

Find the magnitude of the electric flux through the sheet.

$$\Phi_E = EA\cos\phi = \left(13.0\,\frac{\text{N}}{\text{C}}\right)\left(0.105\,\text{m}^2\right)\cos 61.0°$$

$$\approx 0.6825\,\frac{\text{N}\,\text{m}^2}{\text{C}} \approx \boxed{0.682\,\frac{\text{N}\,\text{m}^2}{\text{C}}}$$

Part B

Does the answer to part (A) depend on the shape of the sheet?

So long as the sheet is flat the shape does not matter. $\boxed{No}$.

Part C

For what angle ϕ between the normal to the sheet and the electric field is the magnitude of the flux through the sheet largest?

For a uniform electric field with magnitude E, the electric flux is

$$\Phi_E = EA\cos\phi\,.$$

This is a maximum when $\cos\phi = 1$ or $\phi = 0\,\text{rad} = 0°$. So the electric flux is a maximum when

$$\phi = \boxed{0°}\,.$$

Part D

For what angle ϕ between the normal to the sheet and the electric field is the magnitude of the flux through the sheet smallest?

For a uniform electric field with magnitude E, the electric flux is

$$\Phi_E = EA\cos\phi\,.$$

This is a minimum when $\cos\phi = 0$ or $\phi = \pm\pi = \pm 90°$. So the electric flux is a minium when

$$\phi = \boxed{\pm 90°}\,.$$

22.4 - Electric Flux through a Flat Sheet

A flat sheet is in the shape of a rectangle with sides of length a=0.400m and b=0.600m. The sheet is immersed in a uniform electric field of magnitude E=79.0 N/C that is directed at θ=20° from the plane of the sheet (not the normal).

Part A

Find the magnitude of the electric flux through the sheet.

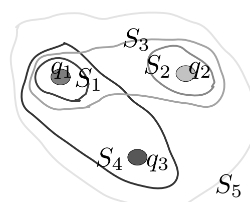
The angle from the electric field to the normal is $\phi = \pi - \theta$.

$$\Phi_E = EA\cos\phi = E\,(a\,b)\cos(\pi - \theta) = EA\sin\theta$$

$$= 79.0\frac{\text{N}}{\text{C}}\ (0.400\text{m}\,0.600\text{m})\sin 20°$$

$$= 6.48470\,\frac{\text{N}\,\text{m}^2}{\text{C}} \approx \boxed{6.48\,\frac{\text{N}\,\text{m}^2}{\text{C}}}$$

22.6 - Electric Flux



Surface	What it encloses
S_1	q_1
S_2	q_2
S_3	q_1 and q_2
S_4	q_1 and q_3
S_5	q_1 and q_2 and q_3

The three small spheres shown in the figure carry charges q_1, q_2, and q_3.

Part A

Find the net electric flux through the closed surface S_1 shown in cross section in the figure.
Use ϵ_0 for the permittivity of free space.

From Gauss's law the electric flux through S_1 is

$$\Phi_E = \oint \vec{E}\cdot \text{d}\vec{A} = \boxed{\frac{q_1}{\epsilon_0}}\,.$$

So you see there is nothing calculated here, it's just a very direct application of Guass's law.

Part B

Find the net electric flux through the closed surface S_2 shown in cross section in the figure.

From Gauss's law the electric flux through S_2 is

$$\Phi_E = \oint \vec{E}\cdot \mathrm{d}\vec{A} = \boxed{\frac{q_2}{\epsilon_0}}$$

Part C

Find the net electric flux through the closed surface S_3 shown in cross section in the figure.

From Gauss's law the electric flux through S_3 is

$$\Phi_E = \oint \vec{E}\cdot \mathrm{d}\vec{A} = \boxed{\frac{q_1+q_2}{\epsilon_0}}$$

Part D

Find the net electric flux through the closed surface S_4 shown in cross section in the figure.

From Gauss's law the electric flux through S_4 is

$$\Phi_E = \oint \vec{E}\cdot \mathrm{d}\vec{A} = \boxed{\frac{q_1+q_3}{\epsilon_0}}$$

Part E

Find the net electric flux through the closed surface S_5 shown in cross section in the figure.

From Gauss's law the electric flux through S_5 is

$$\Phi_E = \oint \vec{E}\cdot \mathrm{d}\vec{A} = \boxed{\frac{q_1+q_2+q_3}{\epsilon_0}}$$

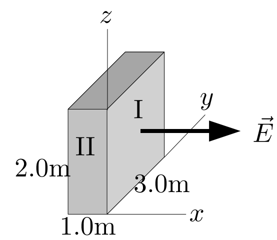
Part F

Do your answers to parts (a) through (e) depend on how the charge is distributed over each small sphere?

Find the net electric flux through a closed surface depends on the total charge enclosed.

answer : $\boxed{\text{do not depend}}$

22.32 - Electric Field



The electric field $\vec{E}$ in the figure is everywhere parallel to the x-axis, so the components E_y and E_z are zero. The x-component of the field E_x depends on x but not on y and z. At points in the yz-plane (where x=0), E_x=125 N/C.

Part A

What is the electric flux, Φ_E, through surface I in the figure?

$$\Phi_E = EA\cos\phi = \left(125\,\frac{\mathrm{N}}{\mathrm{C}}\right)(3\mathrm{m})\,(2\mathrm{m})\cos 0^\circ = \boxed{750\,\frac{\mathrm{N\,m^2}}{\mathrm{C}}}.$$

Part B

What is the electric flux through surface II?

$$\Phi_E = EA\cos\phi = \left(125\,\frac{\mathrm{N}}{\mathrm{C}}\right)(1\mathrm{m})\,(2\mathrm{m})\cos 90^\circ = \boxed{0}.$$

Part C

The volume shown in the figure is a small section of a very large insulating slab 1.0m thick. If there is a total charge -24.0 nC within the volume shown, what is the magnitude of $\vec{E}$ at the face opposite surface I?

We use Gauss' law where the closed Gaussian surface is that which is shown in the figure. There are six flat faces to the Gaussian surface, but only the two faces that are normal to the x-axis have a non-zero controbution to the electric flux through the Gaussian surface. Therefore Gauss' law for this Gaussian surface can be written as

$$\Phi_E(x=0) + \Phi_E(x=-1m) = \frac{Q_{\mathrm{encl}}}{\epsilon_0}$$

$$\Rightarrow \quad \Phi_E(x=-1m) = \frac{Q_{\mathrm{encl}}}{\epsilon_0} - \Phi_E(x=0)$$

$$\Rightarrow \quad -E(x=-1m)A = \frac{Q_{\mathrm{encl}}}{\epsilon_0} - E(x=0)A$$

$$\Rightarrow \quad E(x=-1m) = -\frac{Q_{\mathrm{encl}}}{A\,\epsilon_0} + E(x=0)$$

$$= -\frac{-24\,\mathrm{nC}}{(6\,\mathrm{m}^2)\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{Nm}^2}\right)} + 125\,\frac{\mathrm{N}}{\mathrm{C}}$$

$$\approx 576.9774\,\frac{\mathrm{N}}{\mathrm{C}} \approx \boxed{577\,\frac{\mathrm{N}}{\mathrm{C}}}.$$

Part C

If there is a total charge -24.0 nC within the volume shown, what is the direction of $\vec{E}$ at the face opposite surface I?

We calculated in part A that the x-component of the $\vec{E}$ at the face opposite surface I is 577 N/C. So $\vec{E}$ is in the $+x$ direction.

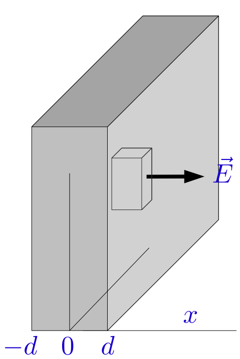
22.54 - Uniformly Charged Slab

A slab of insulating material has thickness $2d$ and is oriented so that its faces are parallel to the yz-plane and given by the planes $x = d$ and $x = -d$. The y and z-dimensions of the slab are very large compared to d and may be treated as essentially infinite. The slab has a uniform positive charge density ρ.

Part A

Using Gauss's law, find the magnitude of the electric field due to the slab at the points $0 \le x \le d$.

From symmetry we argue that the electric field at $x = 0$ is zero and that the direction of the electric field is always in the x direction.



We apply Gauss' law to a Gaussian surface that is a box with one face at $x = 0$ normal to the x axis and a surface area A, and opposite face at x there x is less then d. The electric flux through all surfaces is zero except for this surface.

$$\Phi_E = \oint \vec{E}\cdot \mathrm{d}\vec{A} = \frac{q_{\mathrm{encl}}}{\epsilon_0} \quad\Rightarrow\quad EA = \frac{1}{\epsilon_0}\rho A x$$

$$\Rightarrow \quad E = \boxed{\frac{\rho x}{\epsilon_0}}$$

Part B

What is the direction of the electric field due to the slab at the points $0 \le x \le d$?

$\boxed{+x\text{direction}}$

Part C

Using Gauss's law, find the magnitude of the electric field due to the slab at the points $x \ge d$.

Like in part A, we apply Gauss' law to a Gaussian surface that is a box with one face at $x = 0$ normal to the x axis and a surface area A, and but now the opposite face at x has x is greater d. The electric flux through all surfaces is zero except for this surface.

$$\Phi_E = \oint \vec{E}\cdot \mathrm{d}\vec{A} = \frac{q_{\mathrm{encl}}}{\epsilon_0} \quad\Rightarrow\quad EA = \frac{1}{\epsilon_0}\rho A d$$

$$\Rightarrow \quad E = \boxed{\frac{\rho d}{\epsilon_0}}$$

Part D

What is the direction of the electric field due to the slab at the points $x \ge d$?

$\boxed{+x\text{direction}}$