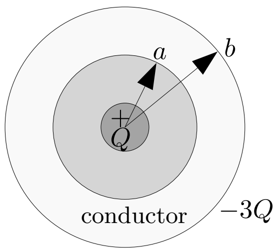
Problem 22.38,42,46; 23.3,17,28 from MasteringPhysics with minor clarifications.

22.38 - Conducting Spherical Shell

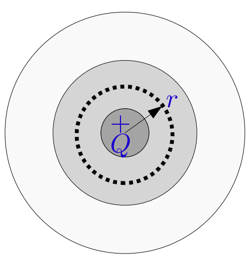


A conducting spherical shell with inner radius a and outer radius b has a positive point charge Q located at its center. The total charge on the shell is -3Q, and it is insulated from its surroundings.

This means the net charge on the conductor is $-3Q$, on both the inner and outer surface.

Part A

Derive the expression for the electric field magnitude in terms of the distance r from the center for the region $r < a$.



We apply Gauss's law to the sphere of radius r shown with the dotted line in the figure above.

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = E \left(4\pi\, r^2\right) = \frac{Q}{\epsilon_0} \quad \Rightarrow \quad E = \boxed{\frac{Q}{4\pi\, \epsilon_0} \frac{1}{r^2}}.$$

Part B

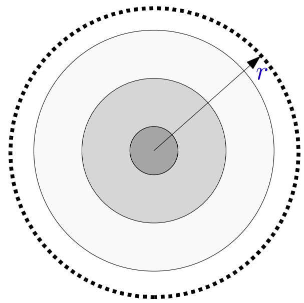
Derive the expression for the electric field magnitude in terms of the distance r from the center for the region $a < r < b$.

The electric field in a conductor is zero. $a < r < b$ has an r value in the conductor. Therefore

$$E = \boxed{0}.$$

Part C

Derive the expression for the electric field magnitude in terms of the distance r from the center for the region $r > b$.



We apply Gauss's law to the sphere of radius r shown with the dotted line in the figure above.

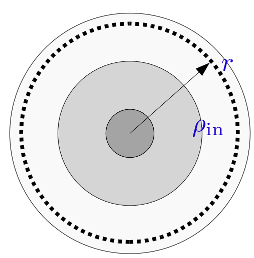
$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = E \left(4\pi\, r^2\right) = \frac{Q - 3Q}{\epsilon_0}\,.$$

Solving for the magnitude of E we get

$$E = \frac{2Q}{4\pi\, \epsilon_0} \frac{1}{r^2} = \boxed{\frac{Q}{2\pi\, \epsilon_0} \frac{1}{r^2}}.$$

Part D

What is the surface charge density on the inner surface of the conducting shell, ρ_{in}?



We apply Gauss's law to the sphere of radius r shown with the dotted line in the figure above.

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = E \left(4\pi\, r^2\right) = \frac{Q + \rho_{\text{in}} \left(4\pi\, a^2\right)}{\epsilon_0}\,.$$

The electric field, E, is zero inside the conductor so this equation can be rewritten as

$$Q + \rho_{\text{in}} \left(4\pi\, a^2\right) = 0 \quad \Rightarrow \quad \rho_{\text{in}} = \boxed{-\frac{Q}{4\pi\, a^2}}.$$

Part E

What is the surface charge density on the outer surface of the conducting shell, $\rho_{\rm out}$?

The charge on the inter surface of the conductor, $q_{\rm in}$, is

$$q_{\rm in} = \rho_{\rm in}\left(4\pi\,a^2\right) = \left(-\frac{Q}{4\pi\,a^2}\right)\left(4\pi\,a^2\right) = -Q$$

The total charge on the conducting shell is given to be $-3Q$.

$$-3Q = q_{\rm out} + q_{\rm in} = q_{\rm out} - Q \quad \Rightarrow \quad q_{\rm out} = -2Q\,.$$

So the charge density (per unit area) on the outer surface of the conductor is

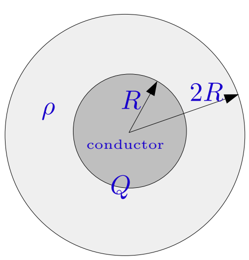
$$\rho_{\rm out} = \frac{q_{\rm out}}{4\pi\,b^2} = -\frac{2Q}{4\pi\,b^2} = \boxed{-\frac{Q}{2\pi\,b^2}}.$$

22.42 - Solid Conducting Sphere with Insulating Shell

A solid conducting sphere with radius R carries a positive total charge Q. The sphere is surrounded by an insulating shell with inner radius R and outer radius $2R$. The insulating shell has a uniform charge density ρ.

Part A

Find the value of ρ so that the net charge of the entire system is zero.



For the total charge to be zero we have

$$0 = Q + \rho\left[\frac{4}{3}\pi\,(2R)^3 - \frac{4}{3}\pi R^3\right] = Q + \rho\frac{4}{3}\pi\,(8-1)\,R^3$$

$$= Q + \rho\frac{28}{3}\pi R^3 \quad \Rightarrow \quad \rho = \boxed{-\frac{3Q}{28\pi R^3}}.$$

Part B

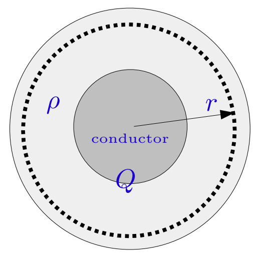
If ρ has the value found in part A, find the magnitude of the electric field, E, in the region $0 < r < R$.

The the region $0 < r < R$ is inside the conductor so E is zero,

$$E = \boxed{0}.$$

Part C

If ρ has the value found in part A, find the magnitude of the electric field in the region $R < r < 2R$.



We apply Gauss's law to the Gaussian surface (dotted line circle) shown in the figure giving

$$\Phi_E = \oint \vec{E}\cdot\vec{A} = E\left(4\pi\,r^2\right) = \frac{Q + \rho\left[\frac{4}{3}\pi\,(r)^3 - \frac{4}{3}\pi R^3\right]}{\epsilon_0}$$

$$= \frac{Q + \frac{4}{3}\rho\pi\left(r^3 - R^3\right)}{\epsilon_0} = \frac{Q + \frac{4}{3}\left(-\frac{3Q}{28\pi R^3}\right)\pi\left(r^3 - R^3\right)}{\epsilon_0}$$

$$= \frac{Q + \left(-\frac{Q}{7R^3}\right)\left(r^3 - R^3\right)}{\epsilon_0} = Q\frac{1 - \frac{1}{7}\left(\frac{r^3}{R^3} - 1\right)}{\epsilon_0}$$

$$= Q\frac{1 - \frac{r^3}{7R^3} + \frac{1}{7}}{\epsilon_0} = \frac{Q}{\epsilon_0}\left(\frac{8}{7} - \frac{r^3}{7R^3}\right) = \frac{8Q}{7\epsilon_0}\left(1 - \frac{r^3}{8\,R^3}\right)$$

$$\Rightarrow \quad E = \frac{1}{4\pi\,r^2}\frac{8Q}{7\epsilon_0}\left(1 - \frac{r^3}{8\,R^3}\right) = \boxed{\frac{2Q}{7\pi\,\epsilon_0\,r^2}\left(1 - \frac{r^3}{8\,R^3}\right)}.$$

Part D

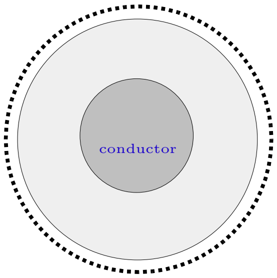
If ρ has the value found in part A, find the direction of the electric field in the region $R < r < 2R$.

For $R < r < 2R$ the value of E in part C is always greater than zero. Therefore the direction of the electric field is always

$$\boxed{\text{radially outward}}.$$

Part E

If ρ has the value found in part A, find the magnitude of the electric field in the region $r > 2R$.



We apply Gauss's law to the Gaussian surface (dotted line circle) shown in the figure. The enclosed charge is zero so

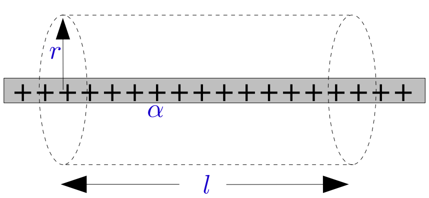
$$\Phi_E = \oint \vec{E} \cdot \vec{A} = E\left(4\pi r^2\right) = \frac{0}{\epsilon_0} \quad \Rightarrow \quad E = \boxed{0}.$$

22.46 - Conducting Tube

A very long conducting tube (hollow cylinder) has inner radius a and outer radius b. It carries charge per unit length $+\alpha$, where alpha is a positive constant with units of C/m. A line of charge lies along the axis of the tube. The line of charge has charge per unit length $+\alpha$.

Part A

Calculate the electric field in terms of α and the distance r from the axis of the tube for $r < a$.



We apply Gauss's law to the Gaussian surface (dotted lines) shown in the figure. The electric flux through sides of the Gaussian cylinder are zero because from symmetry the electric field must be in a radial (direction of increasing r) direction. So Gauss's law gives us

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = E\,(2\pi\, r\, l) = \frac{l\,\alpha}{\epsilon_0}$$

$$\Rightarrow \quad E = \boxed{\frac{\alpha}{2\pi\,\epsilon_0\, r}}.$$

Part B

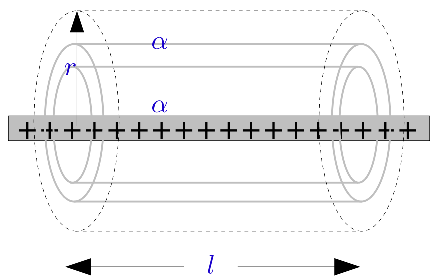
Calculate the electric field in terms of α and the distance r from the axis of the tube for $a < r < b$.

The region with $a < r < b$ is inside of a conductor so

$$E = \boxed{0}.$$

Part C

Calculate the electric field in terms of α and the distance r from the axis of the tube for $r > b$.



We apply Gauss's law to the Gaussian surface (dotted lines) shown in the figure. The electric flux through sides of the Gaussian cylinder are zero because from symmetry the electric field must be in a radial (direction of increasing r) direction. The charge per unit length inclosed will be the charge on the conductor and the charge at the center which adds to 2α. So Gauss's law gives us

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = E\,(2\pi\, r\, l) = \frac{l\,2\alpha}{\epsilon_0}$$

$$\Rightarrow \quad E = \boxed{\frac{\alpha}{\pi\,\epsilon_0\, r}}.$$

Part D

What is the charge per unit length, α_{in}, on the inner surface of the tube?

The tube is a conductor, so inside of the conductor the electric field must be zero, and therefore from Gauss's law the net charge within a Gaussian cylinder that is in the conductor must be zero. So to add to the α at the center there must be a $-\alpha$ on the inside of the conductor.

$$\boxed{-\alpha}.$$

Part E

What is the charge per unit length, α_{out}, on the outer surface of the tube?

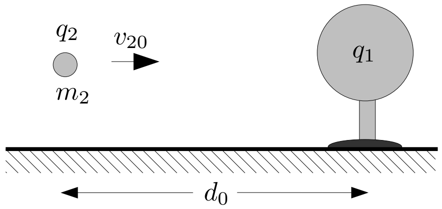
From the statement of the problem we have

$$\alpha = \alpha_{\text{in}} + \alpha_{\text{out}}\,.$$

From part D $\alpha_{\text{in}} = -\alpha$ so

$$\alpha = (-\alpha) + \alpha_{\text{out}} \quad\Rightarrow\quad \alpha_{\text{out}} = \boxed{2\alpha}\,.$$

23.3 - Moving Charges, Energy Methods



A small metal sphere, carrying a net charge of q_1=-3.00μC, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of q_2=-7.30μC and mass m_2=1.70g, is projected toward q_1. When the two spheres are d_0=0.800m apart, q_2 is moving toward q_1 with speed v_{20}=22.0m/s. Assume that the two spheres can be treated as point charges. You can ignore the force of gravity.

Part A

What is the speed, v_{21}, of q_2 when the spheres are d_1=0.430m apart?

We use conservation of energy. We define energy when the distance between the spheres is d_0 as E_0 and the energy when the distance between the spheres is d_1 as E_1.

$$E_0 = E_1 \quad\Rightarrow\quad KE_0 + PE_0 = KE_1 + PE_1$$

$$\Rightarrow\quad \frac{1}{2}m_2\,v_{20}^2 + k\frac{q_1\,q_2}{d_0} = \frac{1}{2}m_2\,v_{21}^2 + k\frac{q_1\,q_2}{d_1} \tag{0.1}$$

$$\Rightarrow\quad \frac{1}{2}m_2\,v_{21}^2 = \frac{1}{2}m_2\,v_{20}^2 + k\frac{q_1\,q_2}{d_0} - k\frac{q_1\,q_2}{d_1}$$

$$\Rightarrow\quad v_{21}^2 = \frac{2}{m_2}\left(\frac{1}{2}m_2\,v_{20}^2 + k\frac{q_1\,q_2}{d_0} - k\frac{q_1\,q_2}{d_1}\right)$$

$$= v_{20}^2 - \frac{2k\,q_1\,q_2}{m_2}\left(\frac{1}{d_1} - \frac{1}{d_0}\right)$$

$$\Rightarrow\quad v_{21} \approx \sqrt{\left(22\frac{\text{m}}{\text{s}}\right)^2 - \frac{2\left(8.988\times10^9\,\frac{\text{Nm}^2}{\text{C}^2}\right)}{1.7\times10^{-3}\text{Kg}} \times (-3\times10^{-6}\,\text{C})\,(-7.3\times10^{-6}\,\text{C})\left[\frac{1}{0.43\text{m}} - \frac{1}{0.8\text{m}}\right]}$$

$$\Rightarrow\quad v_{21} \approx 15.3272\text{m} \approx \boxed{15.3\text{m}}\,.$$

Part B

How close, d_2, does q_2 get to q_1 ?

We can use the same method as in part A, but with $v_{21}\to0$ and $d_1\to d_2$. From equation 0.1 with with $v_{21}\to0$ and $d_1\to d_2$ we have

$$\frac{1}{2}m_2\,v_{20}^2 + k\frac{q_1\,q_2}{d_0} = 0 + k\frac{q_1\,q_2}{d_2}$$

$$\Rightarrow\quad \frac{1}{d_2} = \frac{1}{k\,q_1\,q_2}\left(k\frac{q_1\,q_2}{d_0} + \frac{1}{2}m_2\,v_{20}^2\right)$$

$$\Rightarrow\quad d_2 = k\,q_1\,q_2\frac{1}{k\frac{q_1\,q_2}{d_0} + \frac{1}{2}m_2\,v_{20}^2} = \frac{1}{\frac{1}{d_0} + \frac{m_2\,v_{20}^2}{2k\,q_1\,q_2}}$$

$$\Rightarrow\quad d_2 \approx$$

$$\frac{1}{\frac{1}{0.8\text{m}} + \frac{(1.7\times10^{-3}\,\text{kg})\left(22\,\frac{\text{m}}{\text{s}}\right)^2}{2\left(8.988\times10^9\,\frac{\text{Nm}^2}{\text{C}^2}\right)(-3\times10^{-6}\,\text{C})(-7.3\times10^{-6}\,\text{C})}}$$

$$\approx 0.299397\text{m} \approx \boxed{0.299\text{m}}\,.$$

23.17 - Charge in a Uniform Electric Field

A charge of q is placed in a uniform electric field that is directed vertically upward and that has a magnitude of E.

Part A

What work , W_R, is done by the electric force when the charge moves a distance of x_1 to the right?

$$W_R = \int \vec{F}\cdot\text{d}\vec{r}\,.$$

The force $\vec{F}$ from the electric field is in the upward direction so

$$\vec{F}\cdot\text{d}\vec{r} = 0$$

for the whole displacement to the right. So

$$W_R = \int \vec{F}\cdot\text{d}\vec{r} = \boxed{0}\,.$$

Part B

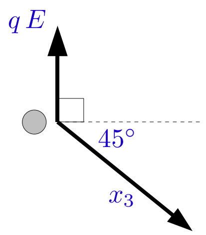
What work, W_U, is done by the electric force when the charge moves a distance of x_2 upward?

In this case the force from the electric field and the displacement are in the same direction so

$$W_U = \int \vec{F} \cdot \mathrm{d}\vec{r} = \boxed{q\,E\,x_2}.$$

Part C

What work, W_{45°, is done by the electric force when the charge moves a distance of x_3 at an angle of 45.0° downward from the horizontal?



$$W_{45^\circ} = \int \vec{F}\cdot\mathrm{d}\vec{r} = \vec{F}\cdot\int \mathrm{d}\vec{r} = q\,\vec{E}\cdot\vec{x}_3 = q\,E\,x_3\cos\left(45^\circ + 90^\circ\right)$$

$$= -q\,E\,x_3\cos 45^\circ = -q\,E\,x_3\cos\frac{\pi}{4} = \boxed{-\frac{q\,E\,x_3}{\sqrt{2}}}.$$

23.28 - Electric Potential

At a certain distance from a point charge, the potential and electric field magnitude due to that charge are $V =$ 4.98V and E=12.0V/m, respectively. (Take the potential to be zero at infinity.)

Part A

What is the distance, d, to the point charge?

Let the unknown charge be q. So we have the two relations

$$E = k\frac{q}{d^2} \quad \text{and} \quad V = k\frac{q}{d}\,. \tag{0.2}$$

Solving for $k\,q$ in both of the above equations we get

$$\Rightarrow \quad E\,d^2 = V\,d \quad \Rightarrow \quad d = \frac{V}{E} \tag{0.3}$$

$$= \frac{4.98\,\mathrm{V}}{12\,\frac{\mathrm{V}}{\mathrm{m}}} = \boxed{0.415\,\mathrm{m}}.$$

Part B

What is the magnitude of the charge, q ?

From equations 0.2 and 0.3 we have

$$V = k\frac{q}{d} = k\frac{q}{\left(\frac{V}{E}\right)} \quad \Rightarrow \quad q = \frac{V^2}{k\,E}$$

$$\approx \frac{(4.98\,\mathrm{V})^2}{\left(8.988\times 10^9\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\right)\left(12\,\frac{\mathrm{V}}{\mathrm{m}}\right)}$$

$$= \frac{\left(4.98\,\frac{\mathrm{Nm}}{\mathrm{C}}\right)^2}{\left(8.988\times 10^9\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\right)\left(12\,\frac{\frac{\mathrm{Nm}}{\mathrm{C}}}{\mathrm{m}}\right)}$$

$$\approx 2.2994\times 10^{-10}\,\mathrm{C} \approx \boxed{2.30\times 10^{-10}\,\mathrm{C}}.$$

Part C

Is the electric field directed toward or away from the point charge?

The potential, V=4.98V, is positive, and so is d.

$$V = k\frac{q}{d}\,,$$

and so q must be positive. For a positive charge the electric field is directed $\boxed{\text{Away}}$ from the charge.