HWIO

Problem 23.38,44,84; 24.12,22 from MasteringPhysics with minor clarifications.

23.38 - parallel conducting plates

Two large parallel conducting plates carrying opposite charges of equal magnitude are separated by L=2.20 cm.

Part A

If the surface charge density for each plate has magnitude 47.0 nC/m^2, what is the magnitude of $\vec{E}$ in the region between the plates?

$$E = \frac{\sigma}{\epsilon_0} = \frac{(47\,\frac{\mathrm{nC}}{\mathrm{m}^2})}{8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\,m}^2}} \approx 5310.73446\,\frac{\mathrm{N}}{\mathrm{C}} \approx \boxed{5311\,\frac{\mathrm{N}}{\mathrm{C}}}$$

Part B

What is the potential difference, ΔV, between the two plates?

The potential difference could be positive or negative, but not enough information was given to determine which. So we'll make ΔV a positive value.

$$\Delta V = -\int_{x=0}^{L} \vec{E}\cdot d\vec{r} = EL \approx \left(5310.73446\,\frac{N}{C}\right)(2.2\times10^{-2}\,\mathrm{m})$$
$$\approx 116.836\,\mathrm{V} \approx \boxed{116.8\,\mathrm{V}}$$

23.44 - metal spheres

A metal sphere with radius r_a is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius r_b. There is charge $+q$ on the inner sphere and charge -q on the outer spherical shell. Take V to be zero when r is infinite.

Part A

Calculate the potential V(r) for $r < r_a$. (Hint: the net potential is the sum of the potentials due to the individual spheres.)

The potential is the sum of the potentials from the two spheres. From fig 23.16 in the text, we see that the potential ~~does not~~ is constant inside a uniformly charged spherical shell. So inside the two spheres

$$V(r) = k\frac{q}{r_a} + k\frac{-q}{r_b} = \boxed{k\,q\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}.$$

Part B

Calculate the potential $V(r)$ for $r_a < r < r_b$.

We are inside one of the spheres and outside the other. The potential for the sphere we are inside constant and the other varies.

$$V(r) = k\frac{q}{r} + k\frac{-q}{r_b} = \boxed{k\,q\left(\frac{1}{r} - \frac{1}{r_b}\right)}.$$

Part C

Calculate the potential $V(r)$ for $r > r_b$.

$$V(r) = k\frac{q}{r} + k\frac{-q}{r} = \boxed{0}.$$

Part D

Find the electric field E, at a point outside the larger sphere at a distance r from the center, where $r > r_b$.

This is a spherically symmetric charge distribution with a net charge of zero, therefore

$$E = \boxed{0}.$$

23.84 - electric potential in space

The electric potential V in a region of space is given by $V(x, y, z) = A(x^2 - 3y^2 + z^2)$ where A is a constant.

Part A

Derive an expression for the electric field $\vec{E}$ at any point in this region.

$$(E_x, E_y, E_z) = \left(-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z},\right) = -A\,(2x, -6y, 2z)$$
$$= \boxed{(-2A\,x, 6A\,y, -2A\,z)}$$

Part B

The work done by the field when a 1.50 μC test charge moves from the point (x, y, z)=(0, 0, 0.250 m) to the origin is measured to be W=6.00 $\times10^{-5}$ J. Determine A.

Note that the electric field does positive work on this charge. The Work, W, is minus the charge in potential energy.

$$W = -\Delta U = -q\,\Delta V$$

$$= -q\left[A(x_2^2 - 3y_2^2 + z_2^2) - A(x_1^2 - 3y_1^2 + z_1^2)\right]$$

$$= -qA\left(0 - z_1^2\right) = qAz_1^2$$

$$\Rightarrow \quad A = \frac{W}{qz_1^2} = \frac{6\times10^{-5}\,\mathrm{J}}{(1.5\times10^{-6}\,\mathrm{C})\,(0.25\,\mathrm{m})^2} = \boxed{640\,\frac{\mathrm{V}}{\mathrm{m}^2}}$$

Part C

Determine the electric field at the point (0, 0, 0.250 m).

$(E_x, E_y, E_z) =$

Part D

In every plane parallel to the xz-plane the equipotential contours are circles. What is the radius of the equipotential contour corresponding to $V = 1280$ V and $y = 2.00$m?

24.12 - spherical capacitor

A spherical capacitor is formed from two concentric, spherical, conducting shells separated by vacuum. The inner sphere has radius R_i=15.0 cm and the capacitance is C=116 pF.

Part A

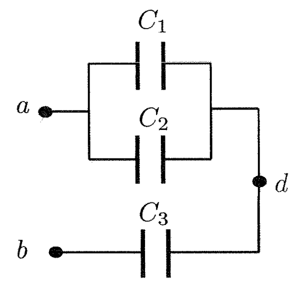
What is the radius of the outer sphere, R_o?

Part B

If the potential difference between the two spheres is V=220 V, what is the magnitude of charge on each sphere?

24.22 - capacitor network

$C_1 = 6.00\mu$F, C_2=3.00μF, C_3=5.00μF. The capacitor network is connected to an applied potential V_{ab}. After the charges on the capacitors have reached their final values, the charge on C_2 is Q_2=40.0μC.



Part A

Part B

Part C

23.84 $\vec{E}_{at}$ (0, 0, 0.25m)

c) $(E_x, E_y, E_z) = (-2A(0),\ 6A(0),\ -2A(0.25\,m))$

$$= \left(0,\ 0,\ -2\ 640\,\tfrac{V}{m^2}\,(0.25\,m)\right) = \boxed{\left(0,\ 0,\ -320\,\tfrac{V}{m}\right)}$$

D) $V = A(x^2 - 3y^2 + z^2) \Rightarrow \frac{V}{A} + 3y^2 = x^2 + z^2$

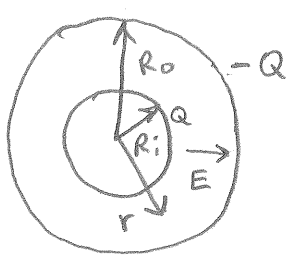
This is the equation of a circle centered at $x=0, z=0$

with radius $\left(\frac{V}{A} + 3y^2\right)^{1/2} = \left(\frac{(1280\,V)}{640\,\frac{V}{m^2}} + 3(2m)^2\right)^{1/2}$

$$= \sqrt{14}\ m \simeq \boxed{3.74\ m}$$

24.12 spherical capacitor

A)



$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{r} = -\int_{R_o}^{R_i} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dr$$

$$= -\left[\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\right]\Bigg|_{R_o}^{R_i} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_i} - \frac{1}{R_o}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\epsilon_0}\left(\frac{1}{R_i} - \frac{1}{R_o}\right)} = 4\pi\epsilon_0\left(\frac{R_i R_o}{R_o - R_i}\right)$$

24.12 A) (continued)

$$C = \frac{4\pi\epsilon_0}{\frac{1}{R_i} - \frac{1}{R_0}} \Rightarrow \frac{1}{R_i} - \frac{1}{R_0} = \frac{4\pi\epsilon_0}{C}$$

$$\Rightarrow \frac{1}{R_0} = \frac{1}{R_i} - \frac{4\pi\epsilon_0}{C} \Rightarrow R_0 = \frac{1}{\frac{1}{R_i} - \frac{4\pi\epsilon_0}{C}}$$

$$= \frac{1}{\frac{1}{0.15\,m} - \frac{4\pi\ 8.85\times10^{-12}\ \frac{C^2}{m^2 N}}{116\times10^{-12}\ F}} \simeq 0.175\ m$$

$$\simeq \boxed{17.5\ cm}$$

B)

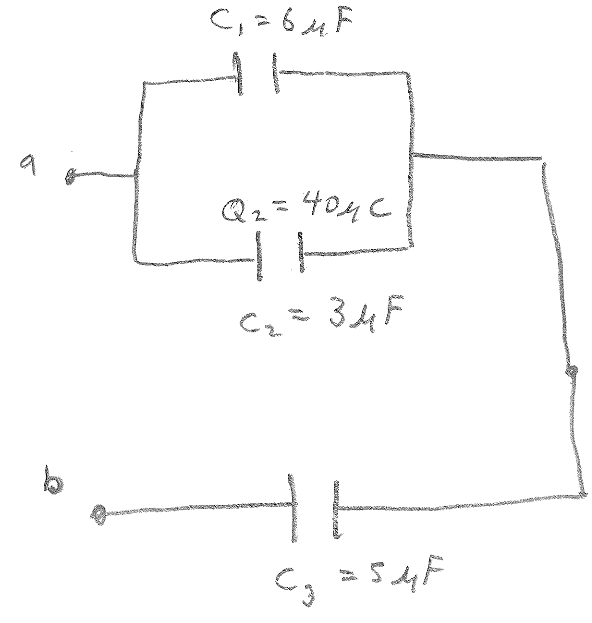
$$C = \frac{Q}{V} \Rightarrow Q = CV = (116\times10^{-12}\ F)(220\ V)$$

$$\simeq \boxed{2.55\times10^{-8}\ C}$$

24.22

A)

V_{ab}



$Q_1 = ?$ $\qquad V_2 = \frac{Q_2}{C_2}$

$$Q_1 = C_1 \underline{V_1} = C_1 \underline{V_2}$$ same V

$$= C_1 \frac{Q_2}{C_2} = \frac{6}{3}\, 40\mu C$$

$$= \boxed{80\mu C}$$

B) If the circuit started out with no charge on it the charge on C_3 would be the same as the charge on C_1 and C_2 combo.

$$C_3 = C_1 + C_3 = 80\mu C + 40\mu C = \boxed{120\mu C}$$

c) $$V_{ab} = \frac{Q}{C_{eq}} = Q\left(\frac{1}{C_1+C_2} + \frac{1}{C_3}\right) = 120\mu C\left(\frac{1}{9\mu F} + \frac{1}{5\mu F}\right)$$

$$\cong \boxed{37.3\ V}$$