Problems 24.28,50,52,74 (24.71) 25.4,20,21,30* from MasteringPhysics (and text).

24.28

A capacitor of capacitance C is charged to a potential difference V_0. The terminals of the charged capacitor are then connected to those of an uncharged capacitor of capacitance $C/2$.

Part A

Compute the original charge of the system.

Part B

Compute the final potential difference across C.

Part C

Compute the final potential difference across $C/2$.

Part D

Compute the final energy of the system.

Part E

Compute the decrease in energy when the capacitors are connected.

24.50

A parallel-plate air capacitor is made by using two plates 16 cm square, spaced 4.7 mm apart. It is connected to a 12-V battery.

Part A

What is the capacitance?

Part B

What is the charge on each plate?

Part C

What is the electric field between the plates?

Part D

What is the energy stored in the capacitor?

Part E

The battery is disconnected and then the plates are pulled apart to a separation of 9.4 mm. What is the capacitance in this case?

Part F

What is the charge on each plate in this case?

Part G

What is the electric field between the plates in this case?

Part H

What is the energy stored in the capacitor?

24.52

Four identical, square, conducting plates have sides of length L. All four plates lie parallel to the xy-plane, with corners at $(x, y) = (L/2, L/2)$, $(- L/2, L/2)$, $(L/2, - L/2)$, and $(- L/2, - L/2)$. Plate 1 is in the plane z=0, plate 2 in the plane z=d, plate 3 in the plane z=2d, and plate 4 in the plane z=3d, where $d \ll L$. Plates 1 and 3 each carry a positive charge Q, and plates 2 and 4 each carry a negative charge - Q. There is vacuum between the plates.

Part A

Find the total energy stored in this arrangement of plates and charges.

Part B

Plates 2 and 3 are now interchanged without altering the charges that they carry. How much work has to be done to carry out this interchange?

24.74

The parallel plates of a capacitor each have an area of $2.00 \times 10^{-1}\mathrm{m}^2$ and are 1.00×10^{-2} m apart. Suppose that the capacitor remains connected to the 3000V power supply while an insulating plastic sheet with $K = 2.50$ is inserted between the plates, completely filling the space between them.

Part A

Compute the magnitude of charge Q on each plate after the dielectric is inserted.

Part B

Compute the magnitude of induced charge Q_i on each face of the dielectric.

Part C

Compute the electric field E after the dielectric is inserted.

Part D

Compute the total energy stored in the electric field after the dielectric is inserted.

Part E

Compute the energy density after the dielectric is inserted.

24.71 - From Text

25.4

A metallic wire has a diameter of 4.12 mm. When the current in the wire is 8.00 A, the drift velocity is 5.40×10^{-5} m/s.

Part A

What is the density of free electrons in the metal?

25.20

A battery-powered light bulb has a tungsten filament. When the switch connecting the bulb to the battery is first turned on and the temperature of the bulb is 20°C, the current in the bulb is 0.860 A. After the bulb has been on for 30 s, the current is 0.220 A.

Part A

What is then the temperature of the filament?

25.21

As part of a lecture demonstration, a physics professor plans to hold an uninsulated current-carrying wire in her hands. For safety's sake, the potential difference between her hands is to be no more than V. She holds her hands separated by a distance of L, with the wire stretched tightly between them. The wire is to carry a current of I and is to be made of aluminum.

Part A

What is the minimum wire radius that is consistent with safety? Take the resistivity of aluminum to be ρ.

25.30

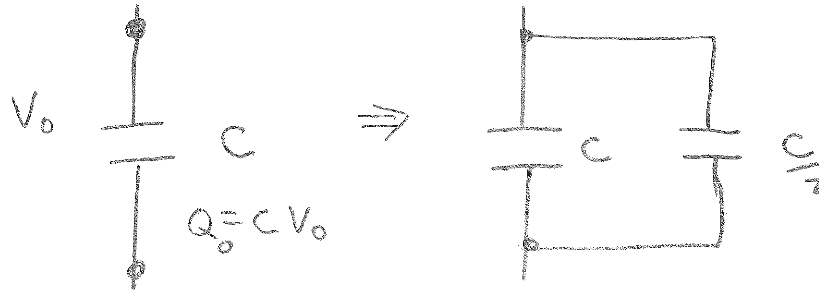
The open-circuit terminal voltage of a battery is 12.6 V. When a resistor R=4.00 Ω is connected between the terminals of the battery, the terminal voltage of the battery is 10.4 V.

Part A

What is the internal resistance of the battery?

24.28

charge now distributed on both C, $\frac{C}{2}$



$C_{eq} = C + \frac{C}{2}$

A) $Q_0 = \boxed{CV_0}$

B) $Q_0 = \left(C + \frac{C}{2}\right) V_f \Rightarrow V_f = \frac{Q_0}{\frac{3}{2}C} = \frac{\cancel{C} V_0}{3/2 \cancel{C}}$

$\Rightarrow V_f = \boxed{\frac{2}{3} V_0}$

C) same $V_f = \boxed{\frac{2}{3} V_0}$

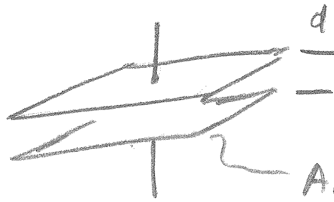
D) $U = \sum \frac{1}{2} CV^2 = \frac{1}{2} C \left(\frac{2}{3} V_0\right)^2 + \frac{1}{2}\left(\frac{C}{2}\right)\left(\frac{2}{3} V_0\right)^2$

$= \left[\frac{\cancel{4}^{\,2}}{\cancel{2}(9)} + \frac{\cancel{4}}{\cancel{4}(9)}\right] CV_0^2 = \frac{3}{9} CV_0^2 = \boxed{\frac{1}{3} CV_0^2}$

E) $U_0 - U_f = \frac{1}{2} CV_0^2 - \frac{1}{3} CV_0^2 = \boxed{\frac{1}{6} CV_0^2}$ decrease

24.50

A)



$$C = \epsilon_0 \frac{A}{d} = 8.85\times10^{-12}\ \frac{C^2}{m^2 N}\ \frac{16\ (0.01m)^2}{0.0047m}$$

$$= 3.013\ pF$$

Wrong They mean $(16\ cm)^2$ = Area

not $16\ cm^2$ = Area

$$C = \frac{\epsilon_0 A}{d} = 8.85\times10^{-12}\ \frac{C^2}{m^2 N}\ \frac{(0.16\,m)^2}{0.0047m} \cong \boxed{48.2\times10^{-12}\ F}$$

B) $Q = CV \cong (48.20425\times10^{-12}\ F)(12\ V) \cong \boxed{5.78\times10^{-10}\ C}$

C) $E = \frac{V}{d} = \frac{12\ V}{4.7\ mm}\ \frac{mm}{10^{-3}m} \simeq \boxed{2550\ \frac{V}{m}}$

D) $U = \frac{1}{2}CV^2 \cong \frac{1}{2}(48.204\ pF)(12\ V)^2$

$\cong \boxed{3.47\times10^{-9}\ J}$

E)

$$C' = C\frac{d}{d'} \cong (4.8204\times10^{-11}\,F)\frac{4.7\,mm}{9.4\,mm}$$

$$\Rightarrow \quad C' \simeq \boxed{2.41\,pF}$$

F) Same. $Q \simeq \boxed{5.78\times10^{-10}\,C}$

G) E is the same $\Rightarrow$ $E = \boxed{2550\,\frac{V}{m}}$

σ = same

H)

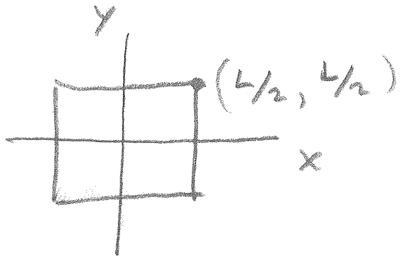
$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2}\frac{1}{\left(\frac{A\epsilon_0}{d}\right)}$$

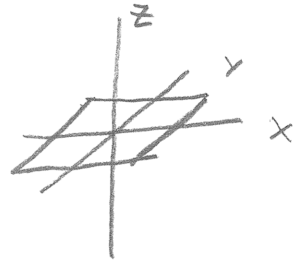
$$\Rightarrow U \propto d \Rightarrow U' = U\frac{d'}{d} \cong 3.47\times10^{-9}\,J\;\frac{9.4\,mm}{4.7\,mm}$$

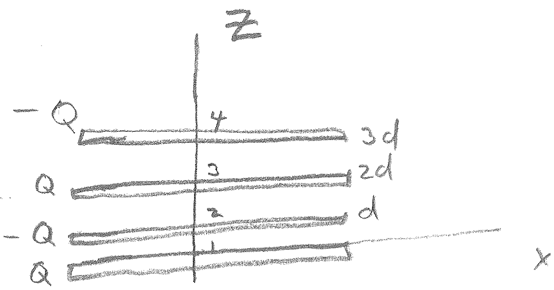
$$\cong \boxed{6.94\times10^{-9}\,J}$$

24.52

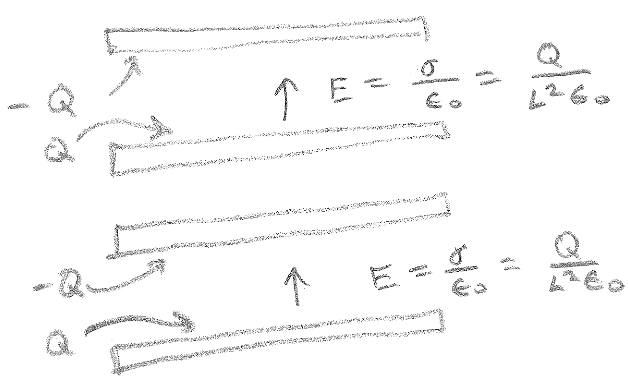
A)







Since the plates conductors there must be no E field in the plates so the charges must be on the surfaces like so.

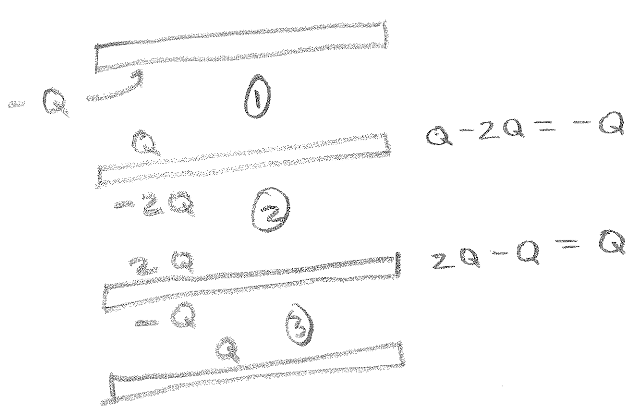


$$\text{Energy} = \left(\tfrac{1}{2}\epsilon_0 E^2\right)(\text{Vol}) + \tfrac{1}{2}\epsilon_0 E^2(\text{vol})$$

$$= \epsilon_0\left(\frac{Q}{L^2\epsilon_0}\right)^2 d L^2$$

$$= \boxed{\frac{Q^2 d}{L^2 \epsilon_0}}$$

B)



The charges must redistubute as shown.

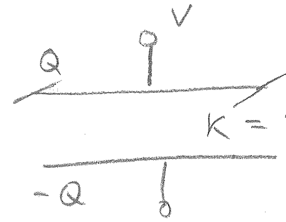
$$\text{Energy} = \overset{①}{\frac{1}{2}\left(\frac{Q^2 d}{L^2\epsilon_0}\right)} + \overset{③}{\frac{1}{2}\,\frac{Q^2 d}{L^2\epsilon_0}} + \overset{②}{\frac{1}{2}\,\frac{(2Q)^2 d}{L^2\epsilon_0}}$$

$$= 3\left(\frac{Q^2 d}{L^2\epsilon_0}\right)$$

$$\Delta\,\text{Energy} = \boxed{\frac{2\,Q^2 d}{L^2\epsilon_0}}$$

24.74

a) $V = 3000\,v$



$A = .2\,m^2$

$d = 10^{-2}\,m$

$K = 2.50$

$C = K C_0 \qquad Q = CV$

$$\Rightarrow Q = K C_0 V = K\left(\epsilon_0 \frac{A}{d}\right) V = 2.50\left(8.85\times10^{-12}\,\frac{F}{m}\right)\frac{.2\,m^2}{10^{-2}\,m}(3000\,v)$$

$$\cong \boxed{1.33\times10^{-6}\ \text{coul}}$$

b)
$$E = \frac{E_0}{K} = \frac{\sigma_0 - \sigma_i}{\epsilon_0} \Rightarrow \frac{E_0}{K} = \frac{\sigma_0}{\epsilon_0} - \frac{\sigma_i}{\epsilon_0} \qquad V = \frac{V_0}{K}$$

$$\Rightarrow \frac{\sigma_i}{\epsilon_0} = \frac{\sigma_0}{\epsilon_0} - \frac{E_0}{K} = E_0\left(1 - \frac{1}{k}\right) \Rightarrow \sigma_i = \epsilon_0\left(\frac{KV}{d}\right)\left(1 - \frac{1}{k}\right)$$

$$\Rightarrow Q_i = \sigma_i A = CV\left(1 - \frac{1}{k}\right) = \boxed{7.96\times10^{-7}\ \text{coul}}$$

c)
$$E = \frac{E_0}{K} = \frac{1}{k}\frac{Q}{\epsilon_0 A}\frac{d}{d} = \frac{Q}{K\frac{\epsilon A}{d}}\frac{1}{d} = \frac{Q}{C}\frac{1}{d} = \frac{\not{C}V}{\not{C}}\frac{1}{d}$$

$$= \frac{V}{d} = \boxed{3\times10^5\ \frac{V}{m}}$$

d)
$$\text{Energy} = \frac{1}{2}CV^2 = \frac{1}{2}QV \cong \frac{1}{2}\left(1.3275\times10^{-6}\,c\right)\left(3000\,\frac{J}{c}\right)$$

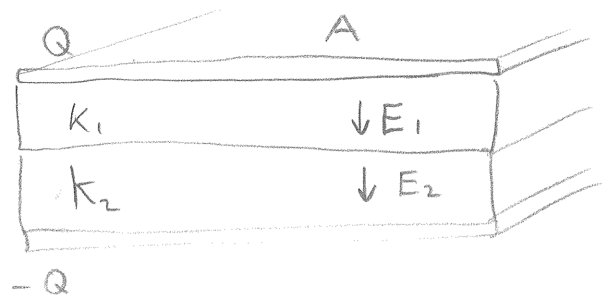
$$\cong \boxed{1.99\times10^{-3}\ J}$$

Energy Density

e) $U = \frac{\text{Energy}}{\text{Volume}} = \frac{1.99125 \times 10^{-3}\,J}{(.2\,m^2)(10^{-2}\,m)} \simeq \boxed{9.96 \times 10^{-1}\,\frac{J}{m^2}}$

f) Energy is added from the constant voltage source.

24.71



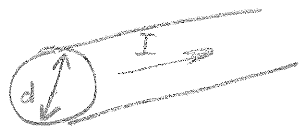
$E_1 = \frac{E_0}{K_1} = \frac{\left(\frac{Q}{\epsilon_0 A}\right)}{K_1} = \frac{Q}{K_1 \epsilon_0 A}$ $\qquad E_2 = \frac{Q}{K_2 \epsilon_0 A}$

$V = E_1\, d/2 + E_2\, \frac{d}{2} = \frac{Q}{K_1 \epsilon_0 A}\frac{d}{2} + \frac{Q}{K_2 \epsilon_0 A}\frac{d}{2}$

$\Rightarrow C = \frac{Q}{V} = \frac{Q}{\left(\frac{1}{K_1} + \frac{1}{K_2}\right)\frac{Q}{\epsilon_0 A}\frac{d}{2}} = \frac{2\epsilon_0 A}{d}\left(\frac{1}{\frac{1}{K_1} + \frac{1}{K_2}}\right)\frac{K_1 K_2}{K_1 K_2}$

$= \frac{2\epsilon_0 A}{d}\frac{K_1 K_2}{K_1 + K_2}$ QED

25.4



$d = 4.12\,mm \qquad I = 8\,A$

$v_d = 5.4\times10^{-5}\,m/s$

$$J = nqv_d \Rightarrow n = \frac{J}{qv_d} = \frac{I}{\frac{\pi d^2}{4}}\frac{1}{qv_d}$$

$$= \frac{8A\ \frac{C}{s}}{\pi\,(4.12\,mm)^2}\ \frac{4}{(1.6\times10^{-19}C)\ 5.4\times10^{-5}m/s}\ \frac{(1\,mm)^2}{(10^{-3}m)^2}$$

$$= \boxed{6.94\times10^{28}\ \frac{1}{m^3}}$$

25.20

$$V = RI = \frac{\rho L}{A} I \Rightarrow \rho = \frac{VA}{L}\frac{1}{I}$$

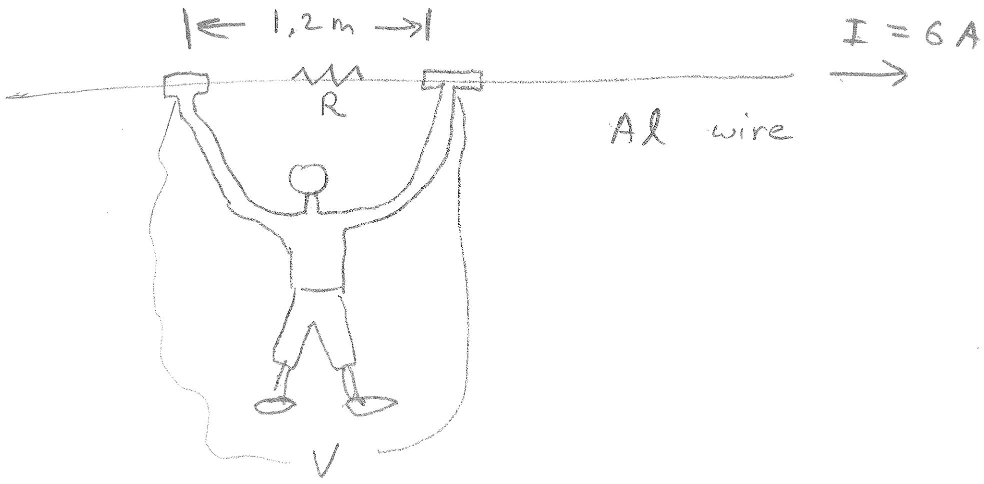
$$\Rightarrow \frac{\rho}{\rho_0} = \frac{I_0}{I} \qquad \rho(T) = \rho_0\left[1 + \alpha(T - T_0)\right]$$

$$\Rightarrow \frac{\rho}{\rho_0} = \frac{I_0}{I} = 1 + \alpha(T - T_0) \Rightarrow T - T_0 = \left(\frac{I_0}{I} - 1\right)\frac{1}{\alpha}$$

$$\Rightarrow T = T_0 + \left(\frac{I_0}{I} - 1\right)\frac{1}{\alpha} = 20^\circ C + \left(\frac{.86\,A}{.22\,A} - 1\right)\frac{1}{0.0045}$$

$$\cong \boxed{666\ ^\circ C}$$

25.21



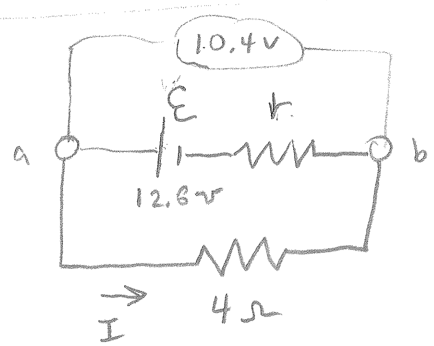
$$V = IR = I\left(\rho \frac{L}{A}\right) = I \frac{\rho L}{(\pi r^2)} \Rightarrow r^2 = \frac{I \rho L}{\pi V}$$

$$\Rightarrow r = \sqrt{\frac{6A \quad 2.75 \times 10^{-8} \left(\frac{V}{A} m\right) (1.2\,m)}{\pi \quad 1.50\,V}}$$

$$= \boxed{0.205\ mm}$$

a thin wire

25.30



$$V_{ab} = \varepsilon - Ir \qquad I = \frac{V_{ab}}{R}$$

$$\Rightarrow r = \frac{\varepsilon - V_{ab}}{I} = \frac{\varepsilon - V_{ab}}{V_{ab}} R$$

$$\Rightarrow r = \frac{\varepsilon - V_{ab}}{V_{ab}} R = \left(\frac{12.6 - 10.4}{10.4}\right)(4\,\Omega) = \boxed{0.846\ \Omega}$$