Problems from 26.30,34,42,58,60; 26.66,83,84,86,88 MasteringPhysics.

26.30

A battery with a voltage of V has an internal resistance of r.

Part A

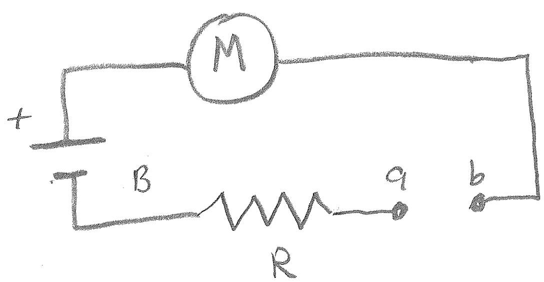
What is the reading of a voltmeter having a resistance of R_V when placed across the terminals of the battery?

Part B

What maximum value may the ratio r/R_V have if the percent error in the reading of the emf of a battery is not to exceed ~~/alpha~~? α

26.34

In the ohmmeter in the figure View Figure , M is a 2.50mA meter of resistance 65.04Ω . (A 2.50mA meter deflects full scale when the current through it is 2.50mA.) The battery B has an emf of 1.52V and negligible internal resistance. R is so chosen that when the terminals a and b are shorted (R_x=0), the meter reads full scale. When a and b are open (R_x=∞), the meter reads zero.



Part A

What is the resistance of the resistor R?

Part B

What current indicates a resistance R_x of 200Ω?

Part C

What value of R_x corresponds to the meter deflection of $\frac{1}{4}$ of full scale if the deflection is proportional to the current through the galvanometer?

Part D

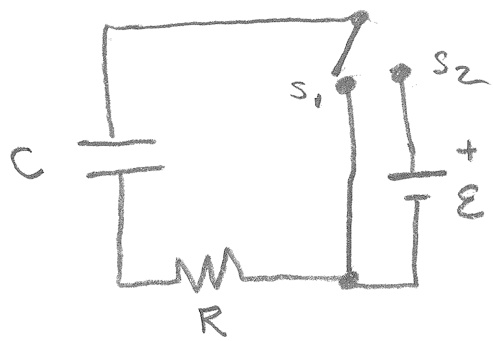
What value of R_x corresponds to the meter deflection of $\frac{1}{2}$ of full scale?

Part E

What value of R_x corresponds to the meter deflection of $\frac{3}{4}$ of full scale?

26.42

In the circuit shown in the figure View Figure , C=5.90μF, $\mathcal{E}$=28.0V, and the emf has negligible resistance. Initially the capacitor is uncharged and the switch S is in position 1. The switch is then moved to position 2, so that the capacitor begins to charge.



Part A

What will be the charge on the capacitor a long time after the switch is moved to position 2?

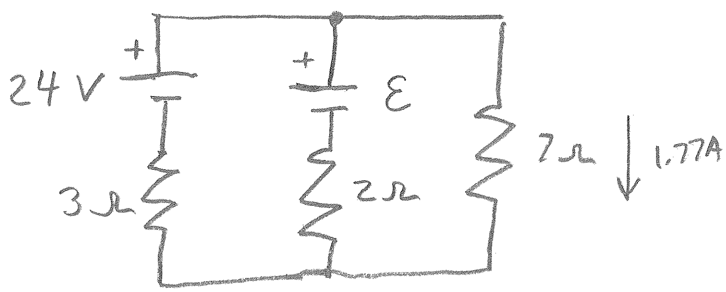
Part B

After the switch has been in position 2 for 3.00 ms, the charge on the capacitor is measured to be 110μC. What is the value of the resistance R?

Part C

How long after the switch is moved to position 2 will the charge on the capacitor be equal to 99.0% of the final value found in part (a)?

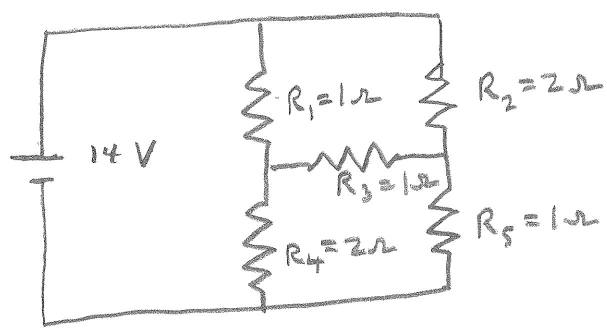
26.58



Part A

What must the emf EMF in the figure View Figure be in order for the current through the 7.00Ω resistor to be 1.77A? Each emf source has negligible internal resistance.

26.60



Part A

Find the current through the battery in the circuit shown in the figure,

Part B

Find the current through the resistor R_1 in the circuit.

Part C

Find the current through the resistor R_2 in the circuit.

Part D

Find the current through the resistor R_3 in the circuit.

Part E

Find the current through the resistor R_4 in the circuit.

Part F

Find the current through the resistor R_5 in the circuit.

Part G

What is the equivalent resistance of the resistor network?

26.66

Three identical resistors are connected in series. When a certain potential difference is applied across the combination, the total power dissipated is P_0.

Part A

What power would be dissipated if the three resistors were connected in parallel across the same potential difference?

26.83

A capacitor that is initially uncharged is connected in series with a resistor and an emf source with negligible internal resistance which provides an EMF of $\mathcal{E}$. Just after the circuit is completed the current through the resistor is I. The time constant for the circuit is t.

Part A

What is the resistance of the resistor?

Part B

What is the capacitance of the capacitor?

26.84

A resistor with a resistance of R is connected to the plates of a charged capacitor with a capacitance of C. Just before the connection is made, the charge on the capacitor is q.

Part A

What is the energy initially stored in the capacitor?

Part B

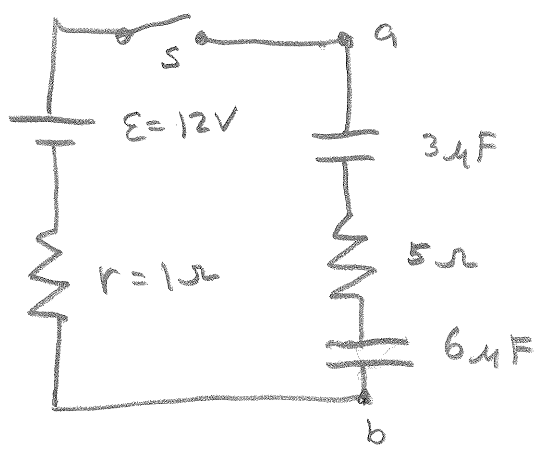
What is the electrical power dissipated in the resistor just after the connection is made?

Part C

What is the electrical power dissipated in the resistor at the instant when the energy stored in the capacitor has decreased to half the value calculated in part (A)?

26.86

Two capacitors in series are charged by a 12.0V battery that has an internal resistance of 1.00Ω . There is a 5.00Ω resistor in series between the capacitors



Part A

What is the time constant of the charging circuit?

Part B

After the switch has been closed for the time determined in part (A), what is the voltage across the 3.00μF capacitor?

26.88

Part A

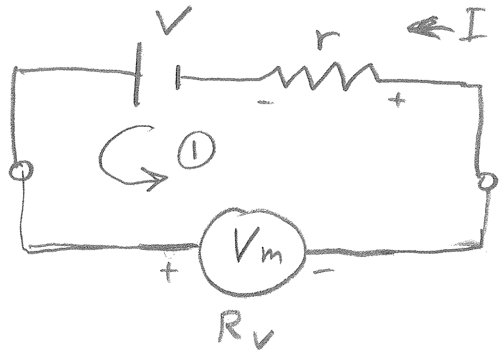
Using equation $i = -\frac{Q_0}{RC}e^{-t/RC}$ for the current in a discharging capacitor, derive an expression for the instantaneous power $P = i^2R$ dissipated in the resistor.

Part B

Integrate the expression for P to find the total energy dissipated in the resistor.

26.30

A)



$$KVR\ ① \Rightarrow V - V_m - Ir = 0$$

$$\Rightarrow V_m = V - Ir$$

$$V_m = I R_V \Rightarrow I(R_V + r) = V \Rightarrow I = \frac{V}{R_V + r}$$

$$\Rightarrow V_m = V - \frac{V}{R_V + r} r = V\left(1 - \frac{r}{R_V + r}\right) = \boxed{V\left(\frac{R_V}{R_V + r}\right)}$$

B)

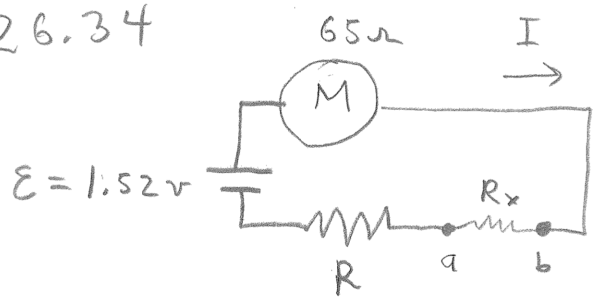
$$\%\ error = \frac{V - V_m}{V} = \frac{V - V\left(\frac{R_V}{R_V + r}\right)}{V}$$

$$= 1 - \frac{R_V}{R_V + r} = \frac{r}{R_V + r} = \frac{r/R_V}{1 + r/R_V}$$

Let $\alpha = \%\ error = \dfrac{(r/R_V)_{max}}{1 + (r/R_V)_{max}} \Rightarrow \left[1 + \left(\dfrac{r}{R_V}\right)_{max}\right]\alpha = \left(\dfrac{r}{R_V}\right)_{max}$

$$\Rightarrow \alpha = \frac{r}{R_V}(1 - \alpha) \Rightarrow \left(\frac{r}{R_V}\right)_{max} = \boxed{\frac{\alpha}{1 - \alpha}}$$

26.34



A)

full scale

$$I = \frac{\varepsilon}{65\Omega + R} \Rightarrow 65\Omega + R = \frac{\varepsilon}{I} \Rightarrow R = \frac{\varepsilon}{I} - 65\Omega$$

$$\Rightarrow R = \frac{1.52\,V}{2.5\times10^{-3}\,A} - 65\Omega = \boxed{543\Omega}$$

B)

$$I = \frac{\varepsilon}{65\Omega + R + R_x} = \frac{1.52\,v}{65\Omega + 543\Omega + 200\Omega} \cong \boxed{1.88\ mA}$$

c)

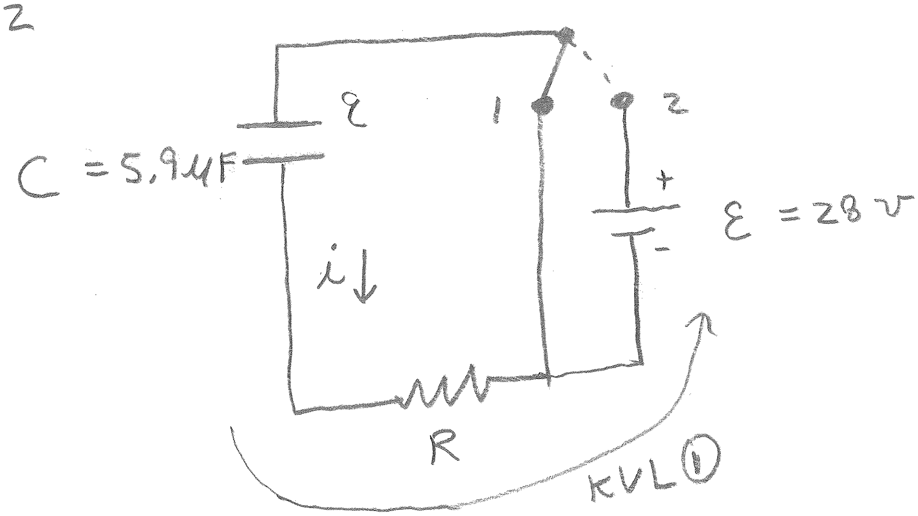
$$65\Omega + R + R_x = \frac{\varepsilon}{I} \Rightarrow R_x = \frac{\varepsilon}{I} - 65\Omega - R =$$

$$R_{x\,\frac{1}{4}} = \frac{1.54\,v}{\frac{2.5\times10^{-3}\,A}{4}} - 608\Omega = 608\Omega(4) - 608\Omega = \boxed{1824\Omega}$$

D) $$R_{x\,\frac{1}{2}} = 608\Omega(2) - 608\Omega = \boxed{608\Omega}$$

E) $$R_{x\,3/4} = 608\Omega\left(\frac{4}{3}\right) - 608\Omega = \boxed{202.\overline{6}\,\Omega}$$

26.42



a)

$$\text{KVL ①} \Rightarrow \mathcal{E} - RI - V_c = 0$$

$$I = 0 \text{ at } t = \infty \Rightarrow V_c = \mathcal{E} \Rightarrow Q = CV_c = C\mathcal{E}$$

$$\Rightarrow Q = (5.9\,\mu F)(28\,v)\frac{10^{-6}}{\mu} = \boxed{1.652 \times 10^{-4} \text{ Coul}}$$

b) $q(t)$, $q(3\,ms) = 110\,\mu C$ $R = ?$

$$\mathcal{E} - iR - V_c = 0 \Rightarrow \mathcal{E} - \frac{dq}{dt}R - \frac{1}{C}q = 0$$

$$\Rightarrow \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} \Rightarrow \int_0^q \frac{dq'}{\frac{\mathcal{E}}{R} - \frac{q'}{RC}} = \int_0^t dt'$$

$$\Rightarrow -RC\int_0^q \frac{dq'}{q' - \mathcal{E}C} = t \Rightarrow (-RC)\left[\ln(q' - \mathcal{E}C)\Big|_0^q\right] = t$$

$$\text{②} \Rightarrow \ln\left(\frac{q - \mathcal{E}C}{-\mathcal{E}C}\right) = -\frac{t}{RC} \Rightarrow q - \mathcal{E}C = -\mathcal{E}C e^{-t/RC}$$

$$\Rightarrow q(t) = \mathcal{E}C(1 - e^{-t/RC})$$

26.42 b (continued)

$$② \Rightarrow R = -\frac{t}{C}\left[\ln\left(1-\frac{q}{\mathcal{E}C}\right)\right]^{-1} = \frac{-3\times10^{-3}\,s}{5.9\times10^{-6}F\ \ln\left(1-\frac{110\,\mu C}{(28\,V)(5.9\,\mu F)}\right)}$$

$$= \boxed{463.8546\ \Omega}$$

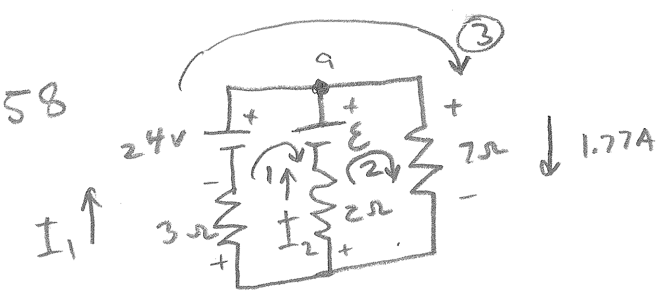
c) $\frac{q(t)}{\mathcal{E}C} = .99 = \left(1-e^{-t/RC}\right) \Rightarrow 1-.99 = e^{-t/RC}$

$$\Rightarrow \ln(0.01) = \frac{-t}{RC} \Rightarrow t = -RC\ \ln(0.01)$$

$$\Rightarrow t = -(463.8546\,\Omega)(5.9\times10^{-6}F)\ \ln(0.01)$$

$$= \boxed{12.6\ ms}$$

26.58



$KCR ⓐ \Rightarrow I_1 + I_2 - 1.77A = 0$

$KVR ① \Rightarrow -I_1(3\Omega) + 24v - \mathcal{E} + I_2(2\Omega) = 0$

$KVR ② \Rightarrow -I_2(2\Omega) + \mathcal{E} - (1.77A)(7\Omega) = 0$

$KVR ③ \Rightarrow 24V - 1.77A(7\Omega) - I_1(3\Omega) = 0$

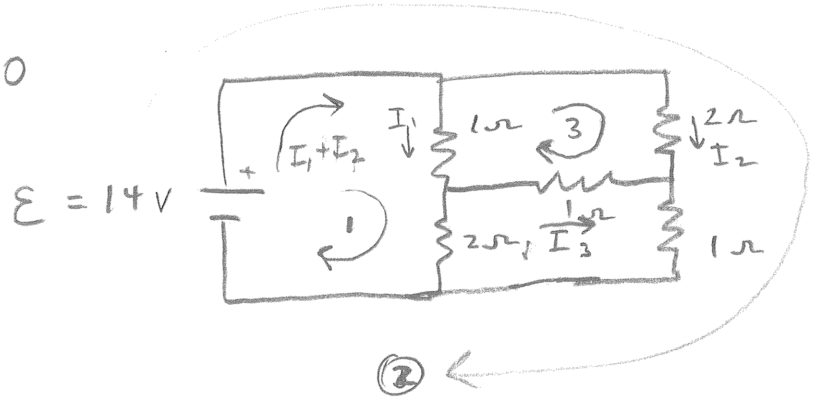
$\Rightarrow I_1 = \frac{24V - 1.77A(7\Omega)}{3\Omega} = 3.87\ A \Rightarrow I_2 = 1.77A - I_1 = 1.77A - 3.87A$

$= -2.1\ A$

$\Rightarrow \mathcal{E} = 1.77A(7\Omega) + I_2(2\Omega) = 1.77A(7\Omega) + (-2.1A)(2\Omega)$

$= \boxed{8.19V}$

26.60



KVL ①

$$\Rightarrow \quad 14\text{ V} - I_1(1\Omega) - (I_1 - I_3)(2\Omega) = 0$$

KVL ②

$$14\text{ V} - I_2(2\Omega) - (I_2 + I_3)(1\Omega) = 0$$

KVL ③

$$-I_2(2\Omega) + I_3(1\Omega) + I_1(1\Omega) = 0$$

3 Equations
3 unknowns

$$① \Rightarrow \quad 3I_1 + 0 - 2I_3 = 14\text{ A}$$

$$② \Rightarrow \quad 0 + 3I_2 + I_3 = 14\text{ A}$$

$$③ \Rightarrow \quad I_1 - 2I_2 + I_3 = 0$$

$$\Rightarrow \begin{array}{cc} \left(\begin{array}{ccc|c} 3 & 0 & -2 & 14 \\ 0 & 3 & 1 & 14 \\ 1 & -2 & 1 & 0 \end{array}\right) & \begin{array}{l} \leftarrow R1 \\ \leftarrow R2 \\ \leftarrow R3 \end{array} \end{array}$$

26.60 (continued)

$$\xrightarrow{R1 - 3R3} \left(\begin{array}{ccc|c} 3 & 0 & -2 & 14 \\ 0 & 3 & 1 & 14 \\ 0 & 6 & -5 & 14 \end{array}\right) \quad 2R2 - R3 \rightarrow \left(\begin{array}{ccc|c} 3 & 0 & -2 & 14 \\ 0 & 3 & 1 & 14 \\ 0 & 0 & 7 & 14 \end{array}\right)$$

$$\Rightarrow I_3 = 2\,A \qquad I_2 = \frac{14-2}{3} = 4\,A \qquad I_1 = \frac{14+2(2)}{3} = 6\,A$$

A)

$$I_1 + I_2 = \boxed{10\,A}$$

B) $\boxed{6\,A}$

C) $\boxed{4\,A}$

D) $\boxed{2\,A}$

E) $I_4 = I_1 - I_3 = (6-2)\,A$

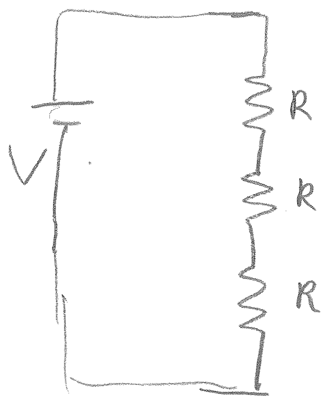
$$= \boxed{4\,A}$$

F) $I_5 = I_2 + I_3 = (4+2)\,A$

$$= \boxed{6\,A}$$

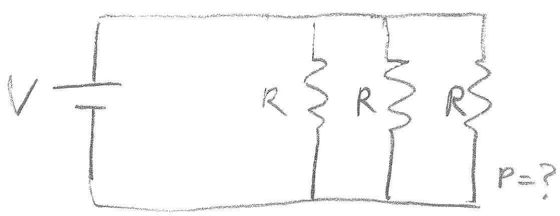
G) $$R_{eq} = \frac{\mathcal{E}}{I_1 + I_2} = \frac{14\,V}{10\,A} = \boxed{1.4\,\Omega}$$

26.66



Power = 27W

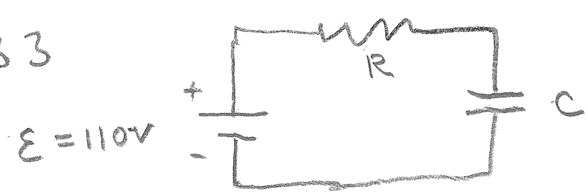
$$\text{Power} = P_o = \frac{V^2}{R_{eq}} = \frac{V^2}{3R} \Rightarrow R = \frac{V^2}{3P_o}$$



$$\frac{1}{R_{eq}} = 3\frac{1}{R} \Rightarrow R_{eq} = \frac{R}{3}$$

$$\Rightarrow P = \frac{V^2}{R_{eq}} = \frac{V^2}{R}3 = \frac{V^2 3}{\left(\frac{V^2}{3P_o}\right)} = 9P_o = 9(27)W = \boxed{243W}$$

26.83



$i(t)$ $\quad i(0) = 6.5 \times 10^{-5} A$

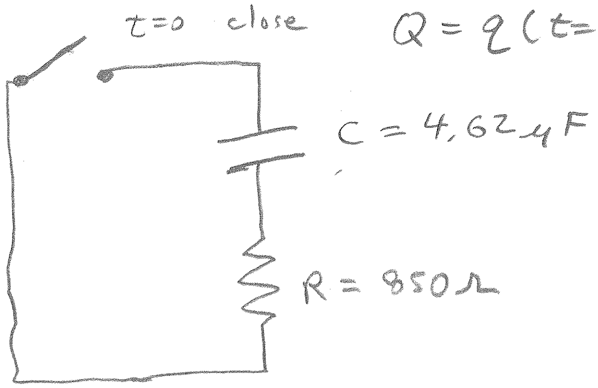
$RC = 6.2\ s$ $\quad R = ?\ C = ?$

at t = 0 the capacitor will act like a short circuit.

$$\Rightarrow i(0) = \frac{\varepsilon}{R} \Rightarrow R = \frac{\varepsilon}{i(0)} = \frac{110V}{6.5 \times 10^{-5} A} = \boxed{1.692\ M\Omega}$$

$$RC = 6.2s \Rightarrow C = \frac{6.2\ s}{1.692 \times 10^6\ \Omega} = \boxed{3.66\ \mu C}$$

26.84



$Q = q(t=0) = 8.10\ mC$

$Q = CV$

$\Rightarrow V = \frac{Q}{C}$

a)

$$\text{Energy} = \frac{1}{2}CV^2 = \frac{1}{2}C\left(\frac{Q}{C}\right)^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\left(\frac{(8.1\ \cancel{m}C)^2}{4.62\ \cancel{\mu}F}\right)$$

$$= \boxed{7.10065\ J}$$

b)

$$P(t=0) = V_R I_R = V_R\left(\frac{V_R}{R}\right) = \frac{V_R^2}{R} = \left(\frac{Q}{C}\right)^2\frac{1}{R}$$

$$= \frac{(8.1\times10^{-3}\ C)^2}{(4.62\times10^{-6}\ F)^2\ (850\ \Omega)} = \boxed{3616.32\ \text{Watts}}$$

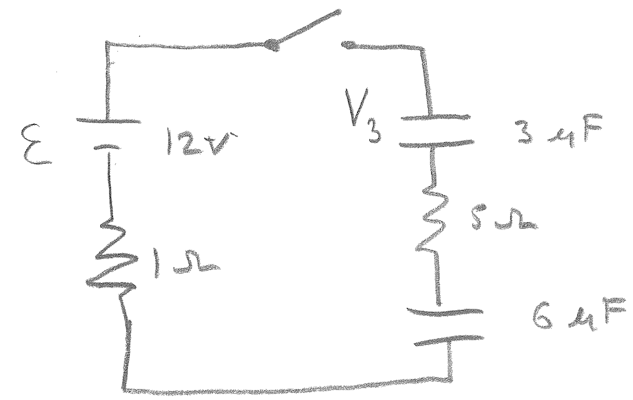
c)

$$P = \frac{1}{2}\left(\frac{Q}{C}\right)^2\frac{1}{R} \qquad (\text{Energy})_{cap} \propto Q^2$$

$$\Rightarrow P \propto (\text{Energy})_{cap} \Rightarrow P' = P\,\frac{(\text{Energy})'_{cap}}{(\text{Energy})_{cap}} = \frac{P}{2}$$

$$\Rightarrow P' = \frac{3616.32\ W}{2} = \boxed{1808.1\ \text{Watts}}$$

26.86



$R_{eq} = 1\Omega + 5\Omega$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

a) $$RC = \left[(1+5)\Omega\right]\left[\frac{3\cdot 6}{3+6}\,\mu F\right] = \boxed{12\,\mu\,\text{sec}}$$

b)

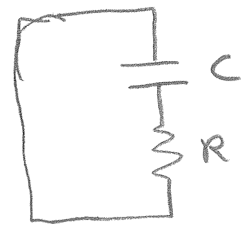
$$q(t) = C_{eq}\,\mathcal{E}\left(1 - e^{-t/RC_{eq}}\right)$$

$$\Rightarrow q(t{=}RC) = C_{eq}\,\mathcal{E}\left(1 - e^{-1}\right) \quad = \text{charge on both caps}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{q(t{=}RC)}{C_3} = \frac{C_{eq}\,\mathcal{E}}{C_3}\left(1 - e^{-1}\right)$$

$$= \left(\frac{\cancel{3}\cdot 6}{3+6}\right)\frac{1}{\cancel{3}}(12\text{ V})\left(1 - e^{-1}\right) = \boxed{5.057\text{ V}}$$

26.88 eq 26.17 $i(t) = I_0 e^{-t/RC}$



$$P_R(t) = i^2 R$$

$$= \boxed{\left(I_0^2 e^{-2t/RC}\right) R = \frac{Q_0^2}{RC^2} e^{-\frac{2t}{RC}}}$$

$$\text{Energy}_{\substack{\text{total} \\ R}} = \int_0^\infty P(t)\,dt = I_0^2 R \int_0^\infty e^{-t/RC}\,dt$$

$$= I_0^2 R\left(-\frac{RC}{2}\right) \int_0^\infty e^{-2t/RC}\, d\left(-\frac{2t}{RC}\right)$$

$$= I_0^2 R\left(-\frac{RC}{2}\right) e^{-2t/RC}\Big|_0^\infty$$

$$= \frac{1}{2} I_0^2 R^2 C \qquad I_0 = \frac{V_c}{R} = \left(\frac{Q}{C}\right)\frac{1}{R} = \frac{Q}{RC}$$

$$= \frac{1}{2}\left(\frac{Q}{RC}\right)^2 R^2 C$$

$$= \boxed{\frac{1}{2}\frac{Q^2}{C}} = \text{Energy from capacitor} \quad \text{QED}$$