HW 16 From Text

27.40 Magnetic Balance. The circuit shown in Fig. 27.46 is used to make a magnetic balance to weigh objects. The mass m to be measured is hung from the center of the bar, that is in a uniform magnetic field of 1.50 T, directed into the plane of the figure. The battery voltage can be adjusted to vary the current in the circuit. The horizontal bar is 60.0 cm long and is made of extremely light-weight material. It is connected to the battery by thin vertical wires that can support no appreciable tension; all the weight of the suspended mass m is supported by the magnetic force on the bar. A resistor with $R = 5.00\ \Omega$ is in series with the bar; the resistance of the rest of the circuit is much less than this. a) Which point, a or b, should be the positive terminal of the battery? b) If the maximum terminal voltage of the battery is 175 V, what is the greatest mass m that this instrument can measure?

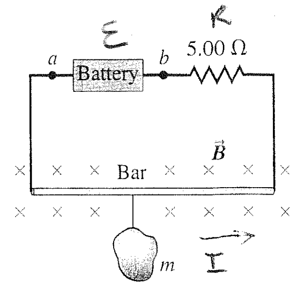
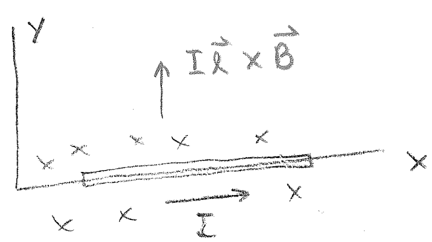


Figure 27.46 Exercise 27.40.

a)

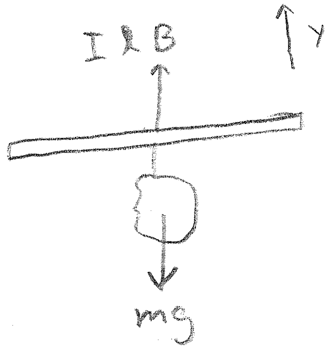


$I\vec{\ell} \times \vec{B} = \ell I B\,\hat{j}$

$\therefore$ I is to the right and $\boxed{\text{a is positive}}$

b)

FBD bar and mass



$\Sigma F_y = I\ell B - mg = 0 \Rightarrow m = \frac{I\ell B}{g}$

$I = \frac{\mathcal{E}}{R} \Rightarrow m = \frac{\mathcal{E}\ell B}{Rg}$

$\Rightarrow m_{max} = \frac{\mathcal{E}_{max}\,\ell B}{R g} = \frac{(175\,V)(.6\,m)(1.5\,T)}{(5\,\Omega)(9.8\,m/s^2)}$

$= \boxed{3.21\ kg}$

27.44 A rectangular coil of wire, 22.0 cm by 35.0 cm and carrying a current of 1.40 A, is oriented with the plane of its loop perpendicular to a uniform 1.50-T magnetic field, as shown in Fig. 27.47. a) Calculate the net force and torque which the magnetic field exerts on the coil.

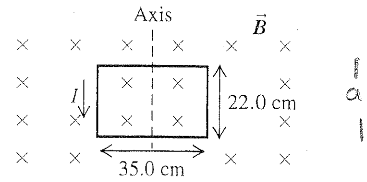
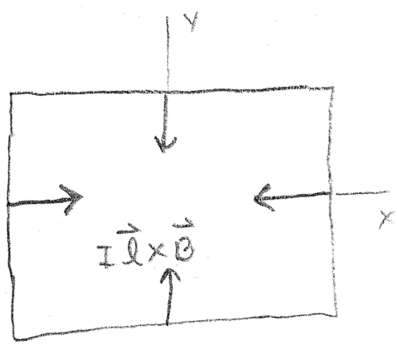


Figure 27.47 Exercise 27.44.

— b —

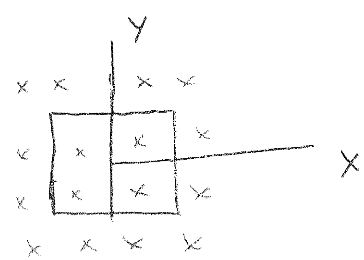
b) The coil is rotated through a 30.0° angle about the axis shown, the left side coming out of the plane of the figure and the right side going into the plane. Calculate the net force and torque which the magnetic field now exerts on the coil. (*Hint:* In order to help visualize this 3-dimensional problem, make a careful drawing of the coil when viewed along the rotation axis.)

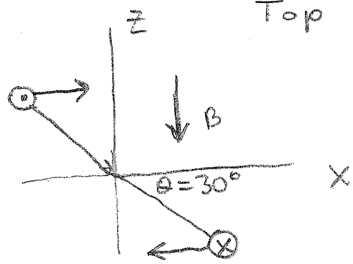
a)



$$\boxed{\vec{F}_{net} = 0 \qquad \vec{\tau}_{net} = 0}$$

b)





$F = I a B$

$$\vec{\tau} = 2(F)\,\frac{b}{2}\sin\theta\ \hat{j}$$

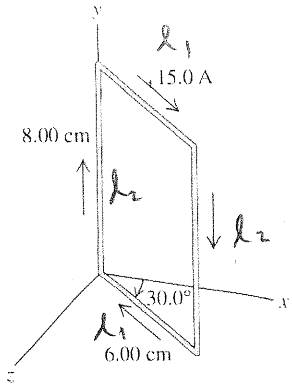
$$= 2(IaB)\,\frac{b}{2}\sin\theta\ \hat{j}$$

$$= I a b B \sin\theta\ \hat{j} = (1.4A)(.22m)(.35m)(1.5T)\sin 30^\circ\ \hat{j}$$

$$= \boxed{8.085\ Nm\ \hat{j}}$$

$\boxed{\vec{F}_{net} = 0}$ for any loop in a uniform $\vec{B}$-field

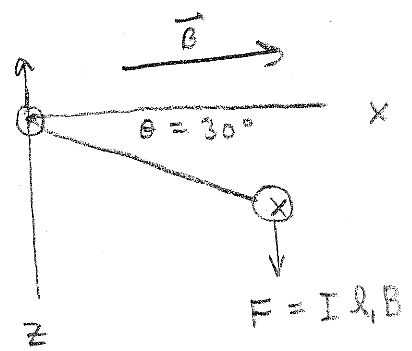
Clockwise from Top View



27.76 The rectangular loop shown in Fig. 27.60 is pivoted about the y-axis and carries a current of 15.0 A in the direction indicated. a) If the loop is in a uniform magnetic field with magnitude 0.48 T in the $+x$-direction, find the magnitude and direction of the torque required to hold the loop in the position shown. b) Repeat part (a) for the case in which the field is in the $-z$-direction. c) For each of the above magnetic fields, what torque would be required if the loop were pivoted about an axis through its center, parallel to the y-axis?

Figure 27.60 Problem 27.76.

a)



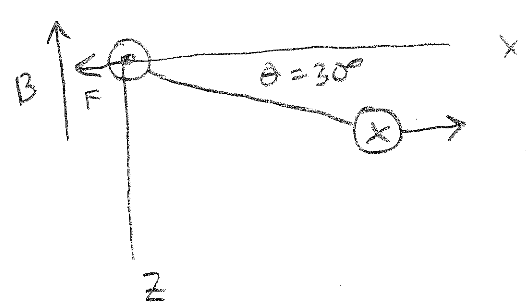
$$\vec{\tau} = (I\ell_1 B)\,\ell_2 \cos 30° \,(-\hat{j})$$

$$= -(15\,A)(.06\,m)(.48\,T)(.08\,m)(\cos 30°)\,\hat{j}$$

$$\Rightarrow \boxed{+0.0299 \text{ Nm } \hat{j}}$$

to hold in place

b)



$$\tau = I\ell_1 B \ell_2 \sin 30° \;\hat{j}$$

$$= (15\,A)(.06\,m)(.48\,T)(.08\,m)(\sin 30°)\,\hat{j}$$

$$\Rightarrow \boxed{-0.0173 \text{ Nm } \hat{j}}$$

to hold in place

c) $\boxed{\text{The same.}}$

The Torque from a couple is always the same about any parallel axis.

27.84 Quark Model of the Neutron. The neutron is a particle with zero charge. Nonetheless, it has a nonzero magnetic moment with z-component 9.66×10^{-27} A·m^2. This can be explained by the internal structure of the neutron. A substantial body of evidence indicates that a neutron is composed of three fundamental particles called *quarks*: an "up" (u) quark, of charge $+2e/3$, and two "down" (d) quarks, each of charge $-e/3$. The combination of the three quarks produces a net charge of $2e/3 - e/3 - e/3 = 0$. If the quarks are in motion, they can produce a nonzero magnetic moment. As a very simple model, suppose the u quark moves in a counterclockwise circular path and the d quarks move in a clockwise, circular path, all of radius r and all with the same speed v (see Fig. 27.64). a) Determine the current due to the circulation of the u quark. b) Determine the magnitude of the magnetic moment due to the circulating u quark. c) Determine the magnitude of the magnetic moment of the three-quark system. (Be careful to use the correct magnetic moment directions.) d) With what speed v must the quarks move if this model is to reproduce the magnetic moment of the neutron? Use $r = 1.20 \times 10^{-15}$ m (the radius of the neutron) for the radius of the orbits.

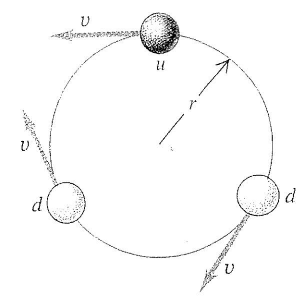


Figure 27.64 Problem 27.84.

a) current $I_u = \dfrac{q_u}{\text{period}} = \dfrac{q_u}{T} = \dfrac{\left(\frac{2}{3}e\right)}{\left(\frac{2\pi r}{v}\right)} = \boxed{\dfrac{1}{3}\dfrac{ev}{\pi r}}$

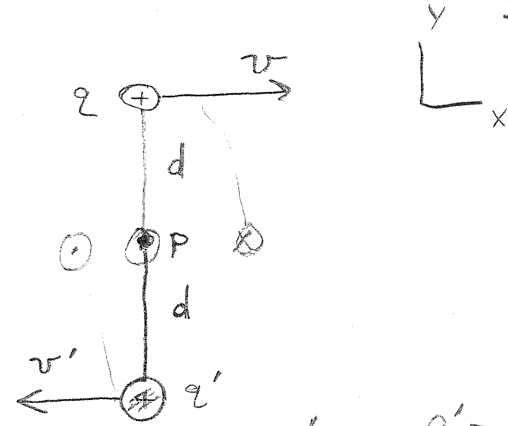
b) $\mu_u = IA = \left(\dfrac{1}{3}\dfrac{ev}{\pi r}\right)(\pi r^2) = \boxed{\dfrac{1}{3}evr}$

c) $\mu_{d_2} = \dfrac{1}{3}evr \qquad \mu_{Total} = \mu_u + \mu_{d_2} = \boxed{\dfrac{2}{3}evr}$

d) $v = \mu \dfrac{3}{2}\dfrac{1}{er} = \dfrac{9.66 \times 10^{-27}\,\text{Am}^2\,(3)}{2\,(1.6 \times 10^{-19}\,\text{C})\,(1.2 \times 10^{-15}\,\text{m})}$

$= \boxed{7.55 \times 10^{7}\ \text{m/s}}$

28.6 Figure 28.29 shows two point charges, q and q', moving relative to an observer at point P. Suppose that the lower charge is actually *negative*, with $q' = -q$. a) Find the magnetic field (magnitude and direction) produced by the two charges at point P if i) $v' = v/2$; ii) $v' = v$; iii) $v' = 2v$. b) Find the direction of the magnetic force that q exerts on q', and find the direction of the magnetic force that q' exerts on q. c) If $v = v' = 3.00 \times 10^5$ m/s, what is the ratio of the magnitude of the magnetic force acting on each charge to that of the Coulomb force acting on each charge?



$q' < 0 \quad q' = -q$

Figure 28.29

a) (i) $v' = v/2$ $\qquad \vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{q\,\vec{v} \times \hat{r}}{r^2}$

$$\vec{B} = \left(-\frac{\mu_0}{4\pi}\frac{qv}{d^2} - \frac{\mu_0}{4\pi}\frac{q'v'}{d^2} \right)\hat{k}$$

$$= \left(-\frac{\mu_0}{4\pi}\frac{qv}{d^2} + \frac{\mu_0}{4\pi}\frac{q}{d^2}\frac{v}{2} \right)\hat{k} = \boxed{-\frac{\mu_0}{8\pi}\frac{qv}{d^2}\hat{k}}$$ into paper

(ii) $$\vec{B} = \left(-\frac{\mu_0}{4\pi}\frac{qv}{d^2} + \frac{\mu_0}{4\pi}\frac{qv}{d^2} \right)\hat{k} = \boxed{0}$$ $v' = v$

(iii) $v' = 2v$

$$\Rightarrow \vec{B} = \left(-\frac{\mu_0}{4\pi}\frac{qv}{d^2} + 2\frac{\mu_0}{4\pi}\frac{qv}{d^2} \right)\hat{k} = \boxed{\frac{\mu_0}{4\pi}\frac{qv}{d^2}\hat{k}}$$ out of paper

b) $$F_{\text{on } q'} = q'\,\vec{v}' \times \vec{B}_q = (-q)\,v'|B_q| = q\,v'|B_q|$$ out of paper

$$= q v' \left[\frac{\mu_0 q v}{4\pi (2d)^2} \right] = \boxed{\frac{q^2 v' v \mu_0}{16\pi d^2} \text{ up}}$$

$$F_{\text{on } q} = q\,\vec{v} \times \vec{B}_{q'} = q v \left[\frac{\mu_0 q v'}{4\pi (2d)^2} \right] = \boxed{\frac{q^2 v' v \mu_0}{16\pi d^2}} \text{ down}$$

28.6 (c)

$$\frac{F_B}{F_e} = \frac{q^2 v' v \mu_0}{16\pi d^2}\left(\frac{4\pi\epsilon_0}{1}\frac{(2d)^2}{q^2}\right)$$

$$= v^2 \mu_0 \epsilon_0$$

$$= \left(3\times10^5\ m/s\right)^2\left(4\pi\times10^{-7}\frac{\cancel{N}\,s^2}{\cancel{C^2}}\right)\left(8.85\times10^{-12}\frac{\cancel{C^2}}{\cancel{N}\,m^2}\right)$$

$$= \boxed{1.00\times10^{-6}}$$

28.12 A long, straight wire lies along the y-axis and carries a current $I = 8.00$ A in the $-y$-direction (Fig. 28.32). In addition to the magnetic field due to the current in the wire, a uniform magnetic field $\vec{B}_0$ with magnitude 1.50×10^{-6} T is in the $+x$-direction What is the total field (magnitude and direction) at the following points in the xz-plane? a) $x = 0$, $z = 1.00$ m; b) $x = 1.00$ m, $z = 0$; c) $x = 0$, $z = -0.25$ m?

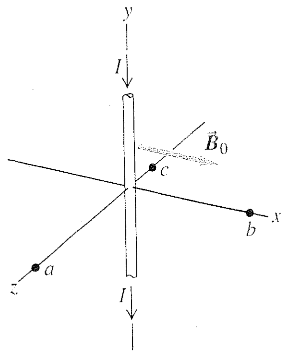


Figure 28.32 Exercise 28.12.

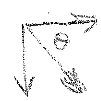
a) $x = 0 \quad z = 1\text{ m}$

$$\vec{B} = -\left(\frac{\mu_0 I}{2\pi r}\right)\hat{i} + B_0\,\hat{i}$$

$$= \left(-\frac{4\pi\times10^{-7}\ \text{Wb/Am}\ (8\text{ A})}{2\pi\ (1\text{ m})} + 1.5\times10^{-6}\text{ T}\right)\hat{i}$$

$$= \boxed{-1.0\times10^{-7}\text{ T}\ \hat{i}}$$

b) $x = 1\text{ m}, \quad z = 0\text{ m}$



$$\vec{B} = 1.5\times10^{-6}\text{ T}\ \hat{i} + \frac{4\pi\times10^{-7}\ \text{Wb/Am}\ (8\text{A})}{2\pi\ (1\text{ m})}\hat{k}$$

$$= 1.5\times10^{-6}\text{ T}\ \hat{i} + 1.6\times10^{-6}\text{ T}\ \hat{k}$$

$$= \sqrt{(1.5)^2 + (1.6)^2}\times10^{-6}\text{ T} \quad \text{in} \quad \tan^{-1}\left(\frac{1.6}{1.5}\right) \text{ for x axis}$$

$$= \boxed{2.19\times10^{-6}\text{ T} \ @\ 46.8^\circ \text{ from x toward z}}$$

c) $$\vec{B} = \left(\frac{4\pi\times10^{-7}\ \text{Wb/Am}\ (8\text{A})}{2\pi\ (.5)\text{ m}} + 1.5\times10^{-6}\text{ T}\right)\hat{i} = \boxed{7.9\times10^{-6}\text{ T}\ \hat{i}}$$

28.19 Four very long, current-carrying wires in the same plane intersect to form a square 40.0 cm on each side, as shown in Fig. 28.35. Find the magnitude and direction of the current I so that the magnetic field at the center of the square is zero.

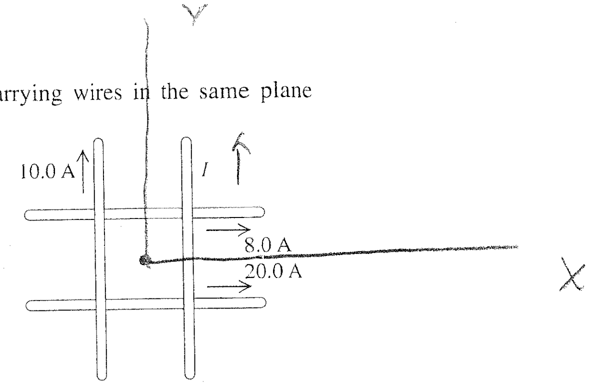


Figure 28.35 Exercise 28.19.

at $(0,0,0)$ $B_x = B_y = 0$

$$B_z = \sum_{i=1}^{4} \frac{\mu_0 I_i}{2\pi r} = \frac{\mu_0}{2\pi r} \sum_i I_i$$

$$\Rightarrow \frac{\mu_0}{2\pi r}\left(-10\,A + I - 8\,A + 20\,A\right) = 0$$

$$\Rightarrow I = -2A \quad \Rightarrow \boxed{I = 2A \text{ down}}$$

28.55 Two long, straight, parallel wires are 1.00 m apart (Fig. 28.46). The upper wire carries a current I_1 of 6.00 A into the plane of the paper. a) What must the magnitude and direction of the current I_2 be for the net field at point P to be zero? b) Then, what are the magnitude and direction of the net field at Q? c) Then, what is the magnitude of the net field at S?

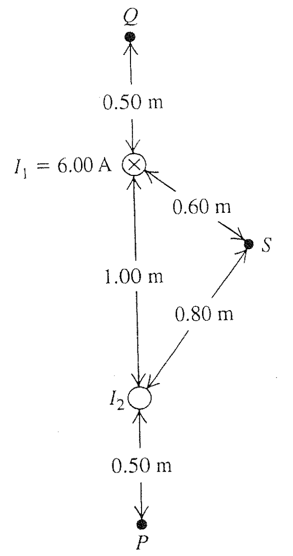
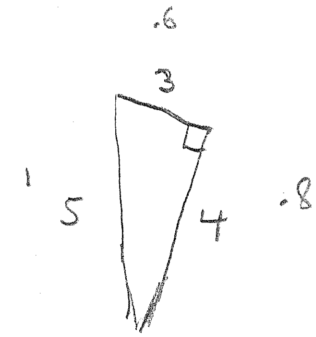


Figure 28.46 Problem 28.55.



$$\sqrt{(.6)^2 + (.8)^2} = 1$$

a)

$$|\vec{B}| = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = 0$$

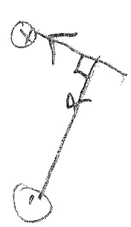
$$\Rightarrow I_2 = I_1 \frac{r_2}{r_1} = 6A\left(\frac{.5}{1.5}\right) = \boxed{2A \text{ out of paper } \odot}$$

b)

$$|\vec{B}|_Q = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = \frac{4\pi \times 10^{-7}\ \text{Wb/Am}}{2\pi}\left(\frac{6A}{.5m} - \frac{2A}{1.5m}\right)$$

$$= \boxed{2.13 \times 10^{-6}\ \text{T} \rightarrow}$$

c)



at right angles

$$|\vec{B}| = \frac{\mu_0}{2\pi}\sqrt{\left(\frac{I_1}{r_1}\right)^2 + \left(\frac{I_2}{r_2}\right)^2}$$

$$= \frac{4\pi \times 10^{-7}\ \text{Wb/Am}}{2\pi}\sqrt{\frac{(6A)^2}{(.6)^2} + \left(\frac{2A}{.8}\right)^2} = \boxed{2.06 \times 10^{-6}\ \text{T}}$$