From Text Book

HW18

1

28.20 Three parallel wires each carry current I in the directions shown in Fig. 28.36. If the separation between adjacent wires is d, calculate the magnitude and direction of the net magnetic force per unit length on each wire.

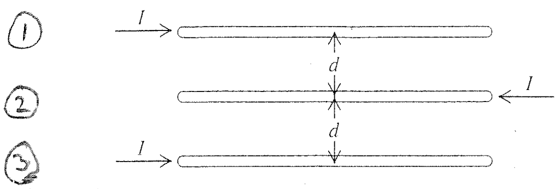
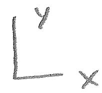


Figure 28.36 Exercise 28.20.



Using

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

Forces are in the y-direction. $F_x = F_z = 0$

Wire

(1) $F_{y_1} = \frac{\mu_0 I^2}{2\pi d} - \frac{\mu_0 I^2}{2\pi(2d)} = \frac{\mu_0 I^2}{4\pi d}$ up

(2) $F_{y_2} = 0$ from symmetry

(3) $F_{y_3} = -\frac{\mu_0 I^2}{4\pi d}$ from symmetry or $\frac{\mu_0 I^2}{4\pi d}$ down

28.22 Two long, parallel wires are separated by a distance of 0.400 m (see Fig. 28.38). The currents I_1 and I_2 have the directions shown. a) Calculate the magnitude of the force exerted by each wire on a 1.20-m length of the other. Is the force attractive or repulsive? b) Each current is doubled, so that I_1 becomes 10.0 A and I_2 becomes 4.00 A. Now what is the magnitude of the force that each wire exerts on a 1.20-m length of the other?

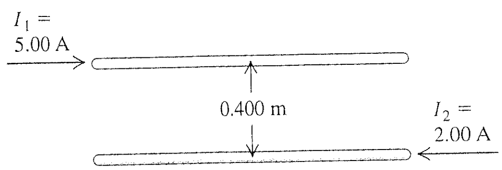


Figure 28.38 Exercise 28.22.

a) The force is repulsive, $\overset{\leftarrow I_1}{\rightarrow I_2}$ opposite.

$$F = L \frac{\mu_0 I I'}{2\pi r} = (1.2\text{ m}) \frac{4\pi \times 10^{-7}\text{ Tm/A}\,(5A)(2A)}{2\pi\,(.4\text{ m})}$$

$$= \boxed{6.0 \times 10^{-6}\text{ N}}$$

b) $$F' = F \frac{(II')_{new}}{(II')_{old}} = F_{(a)} \frac{4}{1} = \boxed{2.4 \times 10^{-5}\text{ N}}$$

28.30 A closed curve encircles several conductors. The line integral $\oint \vec{B} \cdot d\vec{l}$ around this curve is 3.83×10^{-4} T·m. a) What is the net current in the conductors?
b) If you were to integrate around the curve in the opposite direction, what would be the value of the line integral? Explain.

a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow I_{encl} = \frac{1}{\mu_0} \oint \vec{B} \cdot d\vec{l}$

$= \frac{3.83 \times 10^{-4}\ \text{T m}}{4\pi \times 10^{-7}\ \text{T m/A}} = \boxed{304.8\ \text{A}}$

b) In the opposite direction $\vec{B} \cdot d\vec{l} \rightarrow -\vec{B} \cdot d\vec{l}$

$\Rightarrow \oint \vec{B} \cdot d\vec{l} \rightarrow \boxed{-3.83 \times 10^{-4}\ \text{T m}}$

28.32 Coaxial Cable. A solid conductor with radius a is supported by insulating disks on the axis of a conducting tube with inner radius b and outer radius c (see Fig. 28.42). The central conductor and tube carry equal currents I in opposite directions. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude of the magnetic field a) at points outside the central, solid conductor, but inside the tube ($a < r < b$); b) at points outside the tube ($r > c$).

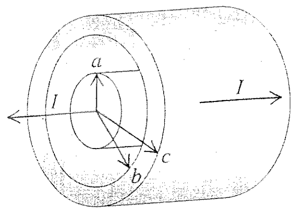
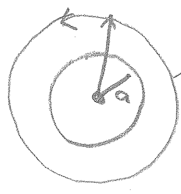


Figure 28.42 Exercises 28.32 and 28.33, and Problem 28.70.

Ampere's Law

A

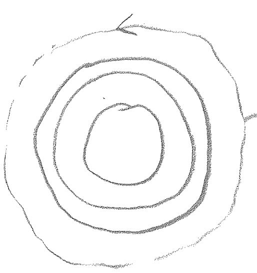
a)



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \quad a < r < b}$$

b)



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} = 0$$

$$\Rightarrow \boxed{B = 0 \quad r > c}$$

28.40

a)

$$B = \frac{\mu_0 N I}{2\pi r} \rightarrow \frac{K_m \mu_0 N I}{2\pi r}$$

$$= \frac{(80)\left(4\pi \times 10^{-7} \frac{T m}{A}\right)(400)(.25 A)}{2\pi (0.06 m)} = \boxed{0.026\bar{6} \text{ T}}$$

b)

$$B = k B_0 \qquad \text{Additional } B = B - B_0$$

$$= B - \frac{B}{K} = B\left(1 - \frac{1}{k}\right) = B\left(\frac{K-1}{K}\right) = B\,\frac{79}{80}$$

$$= \boxed{0.026\bar{3} \text{ T}}$$

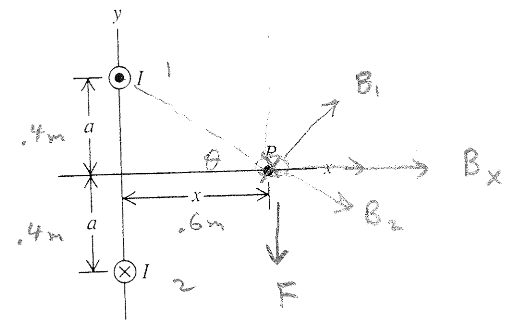


Figure 28.47 Problems 28.56, 28.57, and 28.58.

28.58 Refer to the situation in Problem 28.56. Suppose a third long, straight wire, parallel to the other two, passes through point P (see Fig. 28.47), and that each wire carries a current $I = 6.00$ A. Let $a = 40.0$ cm and $x = 60.0$ cm. Find the magnitude and direction of the force per unit length on the third wire, a) if the current in it is directed into the plane of the figure; b) if the current in it is directed out of the plane of the figure.

$$\vec{B}_{\text{at } P} = B_x \hat{i} = (B_{x1} + B_{x2})\hat{i} = 2 B_{x1} \hat{i}$$

$$B_x = 2\left(\frac{\mu_0 I}{2\pi r}\right)\sin\theta = \frac{\mu_0 I}{\pi r}\frac{a}{r}$$

a)

$$\Rightarrow |\vec{B}| = \frac{\mu_0 I}{\pi r^2} a \qquad |\vec{F}| = I \ell B = \frac{\mu_0 I^2}{\pi r^2} a \ell$$

$$\frac{F}{\ell} = \frac{\mu_0 I^2}{\pi r^2} a = \frac{4\cancel{\pi} \times 10^{-7}\ \text{T m/A}\ (.4\text{m})(6\text{A})^2}{\cancel{\pi}\left[(.4)^2 + (.6)^2\right]\text{m}^2}$$

$$\Rightarrow \frac{\vec{F}}{\ell} = \boxed{1.11 \times 10^{-5}\ \frac{\text{N}}{\text{m}}\ \text{down}}$$

b)

$$\frac{\vec{F}}{\ell} = \boxed{1.11 \times 10^{-5}\ \frac{\text{N}}{\text{m}}\ \text{up}}$$

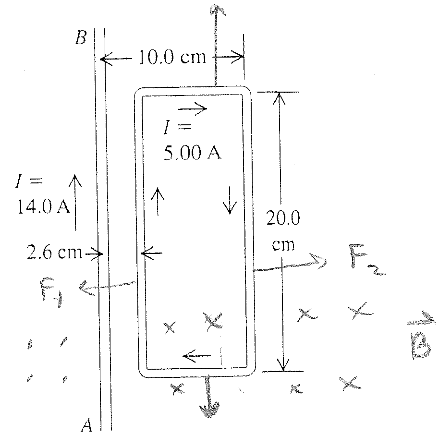
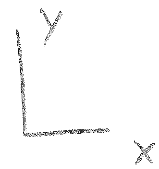


Figure 28.49 Problem 28.62.



$F_2 < F_1$

28.62 The long, straight wire AB shown in Fig. 28.49 carries a current of 14.0 A. The rectangular loop whose long edges are parallel to the wire carries a current of 5.00 A. Find the magnitude and direction of the net force exerted on the loop by the magnetic field of the wire.

$$\vec{F}_{net} = -F_1 \hat{i} + F_2 \hat{i} = \left(-I'\ell B_1 + I'\ell B_2\right)\hat{i}$$

$$= I'\ell\left[-\frac{\mu_0 I}{2\pi r_1} + \frac{\mu_0 I}{2\pi r_2}\right]\hat{i}$$

$$= \frac{\mu_0 I I' \ell}{2\pi}\left[-\frac{1}{r_1} + \frac{1}{r_2}\right]\hat{i}$$

$$= \frac{\cancel{4}^{2}\cancel{\pi} \times 10^{-7}\ \mathrm{Tm/A}\ (5\mathrm{A})(14\mathrm{A})(.2\mathrm{m})}{\cancel{2}\cancel{\pi}}\left[\frac{-1}{(.026\mathrm{m})} + \frac{1}{(.1\mathrm{m})}\right]\hat{i}$$

$$= \boxed{-7.97 \times 10^{-5}\ \mathrm{N}\ \hat{i}}\ \boxed{\text{to the left}}$$

28.66 Calculate the magnitude and direction of the magnetic field produced at point P shown in Fig. 28.52 by the current I in the rectangular wire loop. (Point P is at the center of the rectangle.) (*Hint:* The gap on the left-hand side where the wires enter and leave the rectangle is so small that this side of the rectangle can be taken to be a continuous wire with length b.)

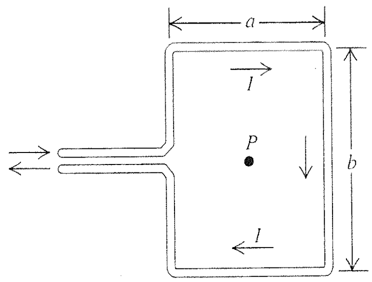
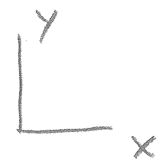


Figure 28.52 Problem 28.66.



Use $\boxed{B_{wire} = \frac{\mu_0 I}{2\pi a}\,\frac{a}{\sqrt{x^2+a^2}}}$

a not the same a

All wire pieces add B-field in the same direction

$$\vec{B} = (2B_a + 2B_b)(-\hat{k}) = \left[\cancel{2}\,\frac{\mu_0 I}{\cancel{2}\pi \frac{b}{\cancel{2}}}\,\frac{\frac{a}{\cancel{2}}}{\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2}} + \cancel{2}\,\frac{\mu_0 I}{\cancel{2}\pi \frac{a}{\cancel{2}}}\,\frac{\frac{b}{\cancel{2}}}{\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2}}\right](-\hat{k})$$

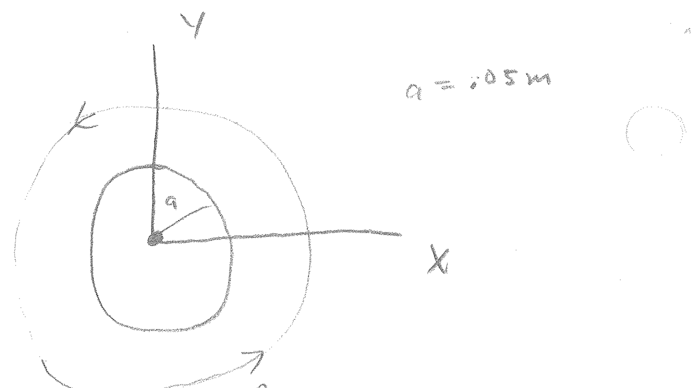
$$|\vec{B}| = \frac{\mu_0 I}{\pi\,\frac{1}{2}\sqrt{a^2+b^2}}\left(\frac{a}{b}+\frac{b}{a}\right) = \frac{2\mu_0 I}{\pi\sqrt{a^2+b^2}}\left(\frac{a}{b}+\frac{b}{a}\right)$$

$$\Rightarrow \quad \vec{B} = \boxed{-\frac{2\mu_0 I}{\pi\sqrt{a^2+b^2}}\left(\frac{a}{b}+\frac{b}{a}\right)\hat{k}}$$

28.76 A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is $\vec{J}$. The current density, although symmetrical about the cylinder axis, is not constant and varies according to the relation

$$\vec{J} = \left(\frac{b}{r}\right)e^{(r-a)/\delta}\hat{k} \quad \text{for} \quad r \le a$$
$$= 0 \quad \text{for} \quad r \ge a$$

where the radius of the cylinder is $a = 5.00$ cm, r is the radial distance from the cylinder axis, b is a constant equal to 600 A/m^{-1}, and δ is a constant equal to 2.50 cm. a) Let I_0 be the total current passing through the entire cross section of the wire. Obtain an expression for I_0 in terms of b, δ, and a. Evaluate your expression to obtain a numerical value for I_0. b) Using Ampere's law, derive an expression for the magnetic field $\vec{B}$ in the region $r \ge a$. Express your answer in terms of I_0 rather than b. c) Obtain an expression for the current I contained in a circular cross section of radius $r \le a$ and centered at the cylinder axis. Express your answer in terms of I_0 rather than b. d) Using Ampere's law, derive an expression for the magnetic field $\vec{B}$ in the region $r \le a$. e) Evaluate the magnitude of the magnetic field at $r = \delta$, $r = a$, and $r = 2a$.



a) $$I = \int \vec{J}\cdot d\vec{A} = \int_0^{r'=a} J(r')\,(2\pi r')\,dr'$$

$$= \int_0^a \frac{b}{\cancel{r'}} e^{(r'-a)/\delta}\, 2\pi \cancel{r'}\, dr = 2\pi b \int_0^a e^{\frac{r}{\delta}-\frac{a}{\delta}}\, dr$$

$$= 2\pi b\, e^{-a/\delta}\, \delta \int_0^{x=a/\delta} e^{x}\, dx$$

$$= 2\pi b\, e^{-a/\delta}\, \delta \left(e^{a/\delta} - 1\right) = \boxed{2\pi b \delta\left[1 - e^{-a/\delta}\right]}$$

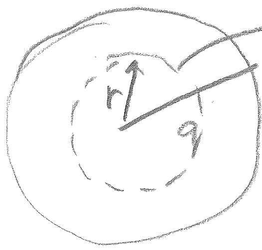
$$= 2\pi\, 600\,\frac{A}{m}\,(.025\,m)\left[1 - e^{-5/2.5}\right]$$

$$= \boxed{81.49273\ A}$$

b) $$\oint \vec{B}\cdot d\vec{A} = B(2\pi r) = \mu_0 I_0 \Rightarrow B = \boxed{\frac{\mu_0 I_0}{2\pi r}}$$

c) $$I(r) = I(a \to r) = \boxed{2\pi b\delta\left[1 - e^{-r/\delta}\right]} = \boxed{I_0 \frac{(1 - e^{-r/\delta})}{(1 - e^{-a/\delta})}}$$

28.76 d)



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\Rightarrow B\, 2\pi r = \mu_0 I(r)$$

$$\Rightarrow B = \frac{\mu_0}{2\pi r} I(r) = \frac{\mu_0}{2\pi r} I_0 \frac{(1-e^{-r/\delta})}{(1-e^{-a/\delta})} \qquad 0 \le r \le a$$

e)

$$B(r=\delta) = \frac{\mu_0}{2\pi\delta} I_0 \frac{1-e^{-1}}{1-e^{-a/\delta}}$$

$$= \frac{4\pi \times 10^{-7}\ \mathrm{Tm/A}}{2\pi(.025)} (81.49273\ \mathrm{A}) \frac{1-e^{-1}}{1-e^{-2}}$$

$$\cong 1.75 \times 10^{-4}\ \mathrm{T}$$

$$B(r=a) = \frac{\mu_0}{2\pi a} I_0 = \frac{\overset{2}{\cancel{4\pi}} \times 10^{-7}\ \mathrm{Tm/A}}{\cancel{2\pi}(.05\ \mathrm{m})} (81.49273\ \mathrm{A})$$

$$= \boxed{3.2597 \times 10^{-4}\ \mathrm{T}}$$

$$B(r=2a) = \frac{\mu_0 I_0}{2\pi(2a)} = \frac{1}{2} B(a) \cong \boxed{1.63 \times 10^{-4}\ \mathrm{T}}$$

28.80 Long, straight conductors with square cross section, each carrying current I, are laid side-by-side to form an infinite current sheet with current directed out of the plane of the page (see Fig. 28.57). A second infinite current sheet is a distance d below the first and is parallel to it. The second sheet carries current into the plane of the page. Each sheet has n conductors per unit length. (Refer to Problem 28.79.) Calculate the magnitude and direction of the net magnetic field at a) point P (above the upper sheet); b) point R (midway between the two sheets); c) point S (below the lower sheet).

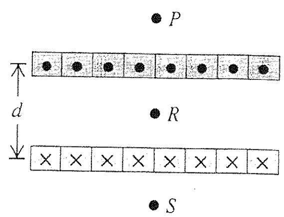
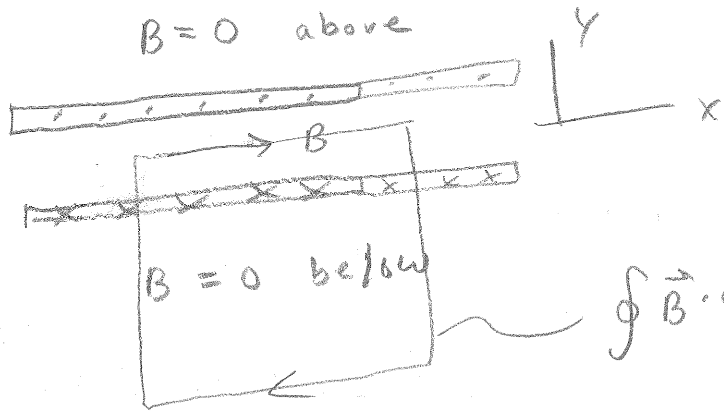


Figure 28.57 Problem 28.80.



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\Rightarrow \quad B_{in} = \mu_0 n I$$

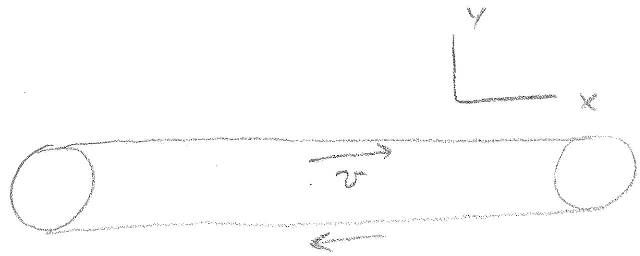
$$\Rightarrow \quad B(l) = \mu_0 n l I$$

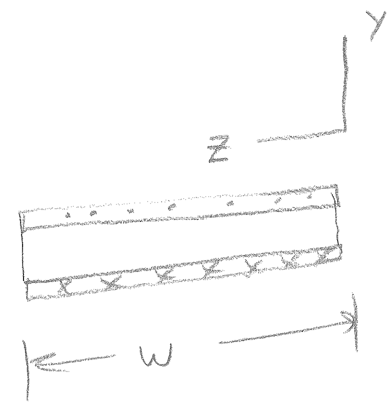
a) $B = 0$

b) $B = \mu_0 n I \, \hat{i}$ uniform

c) $B = 0$

28.84 A wide, long, insulating belt has a uniform positive charge per unit area σ on its upper surface. Rollers at each end move the belt to the right at a constant speed v. Calculate the magnitude and direction of the magnetic field produced by the moving belt at a point just above its surface. (*Hint:* At points near the surface and far from its edges or ends, the moving belt can be considered to be an infinite current sheet like that in Problem 28.79.)

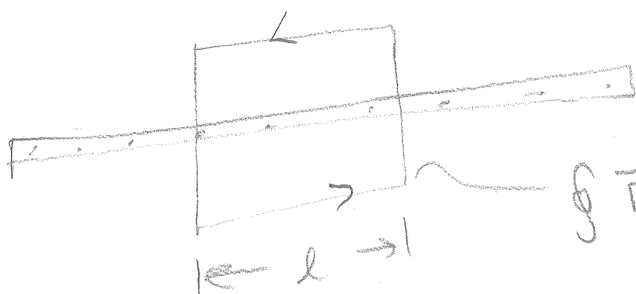




$$I = \sigma w v$$

If we ignore the lower belt

current at you



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\Rightarrow 2Bl = \mu_0 \sigma v l$$

$$\Rightarrow \boxed{B = \frac{\mu_0 \sigma v}{2}}$$