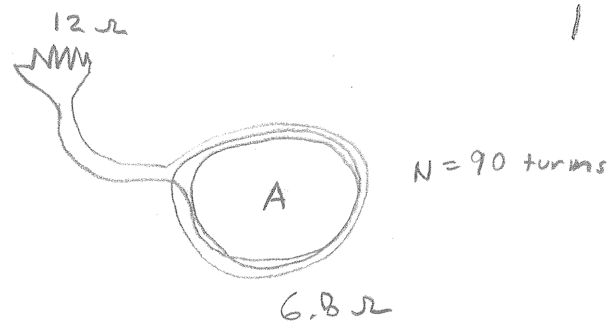
From Text book

HW 19



**29.4** The cross-sectional area of a closely wound search coil (Exercise 29.3) having 90 turns is 2.20 $cm^2$, and its resistance is 6.80 Ω. The coil is connected through leads of negligible resistance to a charge-measuring instrument with an internal resistance of 12.0 Ω. Find the quantity of charge displaced when the coil is pulled quickly out of a region, where $B = 2.05$ T, to a point where the magnetic field is zero. The plane of the coil, when in the field, makes an angle of 90° with the magnetic field.




$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$ Ampere's Law

$$A = 2.2\,cm^2 \times \frac{m^2}{(100\,cm)^2} = 2.2\times10^{-4}\,m^2$$

$$\mathcal{E} = \left(\frac{dq}{dt}\right)R$$ Ohm's Law

$$\Rightarrow \quad \frac{dq}{dt}R = -N\frac{d\Phi_B}{dt} = -NA\frac{dB}{dt}$$

$$Q = \int_0^Q \left(\frac{dq}{dt}\right)dt = -\frac{NA}{R}\int_B^0 \frac{dB'}{dt}\,dt \Rightarrow Q = -\frac{NA}{R}(0-B)$$

$$\Rightarrow Q = \frac{NAB}{R} = (90)\frac{(2.2\times10^{-4}\,m^2)(2.05\,T)}{(12.0\,\Omega + 6.8\,\Omega)}$$

$$= \boxed{2.159\times10^{-3}\ C}$$

2

**29.16** A circular loop of wire is in a region of spatially uniform magnetic field, as shown in Fig. 29.27. The magnetic field is directed into the plane of the figure. Determine the direction (clockwise or counterclockwise) of the induced current in the loop when a) $B$ is increasing; b) $B$ is decreasing; c) $B$ is constant with value $B_0$. Explain your reasoning.

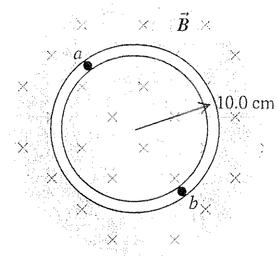



**Figure 29.27** Exercises 29.16, 29.29, and Problem 29.48.

a) Lenz's law say's that the loop with make a B-field that apposes.

Right hand Rule $\Rightarrow$ $\boxed{\text{counter clock wise}}$

b) $\boxed{\text{clock wise}}$

c) No current induced $\frac{d\Phi_B}{dt} = 0$

**29.17** Using Lenz's law, determine the direction of the current in resistor *ab* of Fig. 29.28 when a) switch *S* is opened after having been closed for several minutes; b) coil *B* is brought closer to coil *A* with the switch closed; c) the resistance of *R* is decreased while the switch remains closed.

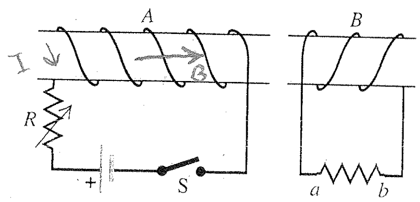



**Figure 29.28** Exercise 29.17.

a) $\boxed{\text{from a to b}}$ B is decreasing, induced B must appose

b) B gets stronger ⇒ $\boxed{\text{from b to a}}$ will appose

c) B gets stronger ⇒ $\boxed{\text{from b to a}}$ will appose

**29.22** In Fig. 29.32 a conducting rod with length $L = 30.0$ cm moves in a magnetic field $\vec{B}$ of magnitude 0.450 T directed into the plane of the figure. The rod moves with speed $v = 5.00$ m/s in the direction shown. a) What is the motional emf induced in the rod? b) What is the potential difference between the ends of the rod? c) Which point, $a$ or $b$, is at higher potential? d) When the charges in the rod are in equilibrium, what are the magnitude and direction of the electric field within the rod? e) When the charges in the rod are in equilibrium, which point, $a$ or $b$, has an excess of positive charge?

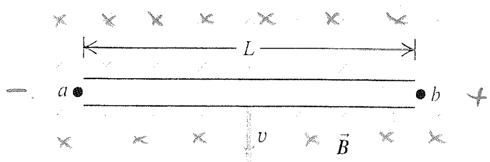



**Figure 29.32** Exercise 29.22.

a) $\mathcal{E} = \vec{v} \times \vec{B} \cdot \vec{L}$ $\qquad \mathcal{E} = vBL$

$= (5\ m/s)(0.45\ T)(0.30\ m) = \boxed{0.675\ \text{Volts}}$

b) $\boxed{0.675\ \text{volts}}$

c) $\boxed{b}$ from $\mathcal{E} = \vec{v} \times \vec{B} \cdot \vec{L}$

d) $E = \dfrac{\mathcal{E}}{L} = vB = (5\ m/s)(0.45\ T) = \boxed{2.25\ \frac{\text{volts}}{m}}$

to the left

e) $\boxed{b}$

**29.24** The conducting rod $ab$ shown in Fig. 29.33 makes contact with metal rails $ca$ and $db$. The apparatus is in a uniform magnetic field 0.800 T, perpendicular to the plane of the figure. a) Find the magnitude of the emf induced in the rod when it is moving toward the right with a speed 7.50 m/s. b) In what direction does the current flow in the rod? c) If the resistance of the circuit $abdc$ is 1.50 Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s. You can ignore friction. d) Compare the rate at which mechanical work is done by the force ($Fv$) with the rate at which thermal energy is developed in the circuit ($I^2R$).

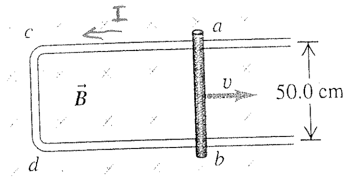



**Figure 29.33** Exercise 29.24.

a) $$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{d\,BA}{dt} = B\frac{dA}{dt} = B l v = (.8T)(0.5m)(7.5\ m/s)$$

$$= \boxed{3 \text{ volts}}$$

b) Lenz's Law $\Rightarrow$ $\boxed{\text{counter clock wise}}$

c) $I = \frac{\mathcal{E}}{R}$   $F = I l B = \frac{\mathcal{E}}{R} l B = \left(\frac{3 \text{ volts}}{1.5\,\Omega}\right)(.5m)(.8T)$

$$= \boxed{.8\ N}$$

d) $$F \cdot v = I l B v = \frac{\mathcal{E}}{R} l B v = \frac{(Blv)(lBv)}{R}$$

$$= \frac{l^2 B^2 v^2}{R} = \frac{\mathcal{E}^2}{R} = \boxed{6 \text{ watts}}$$

$Fv = \frac{\mathcal{E}^2}{R}$   there are the same

**29.54** The current in the long, straight wire $AB$ shown in Fig. 29.39 is upward and is increasing steadily at a rate $di/dt$. a) At an instant when the current is $i$, what are the magnitude and direction of the field $\vec{B}$ at a distance $r$ to the right of the wire? b) What is the flux $d\Phi_B$ through the narrow, shaded strip? c) What is the total flux through the loop? d) What is the induced emf in the loop? e) Evaluate the numerical value of the induced emf if $a = 12.0$ cm, $b = 36.0$ cm, $L = 24.0$ cm, and $di/dt = 9.60$ A/s.

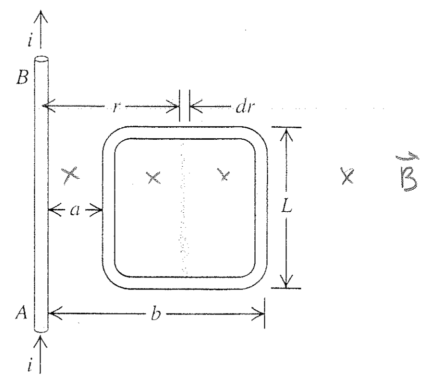



**Figure 29.39** Problem 29.54.

a) $$B = \frac{\mu_0 I}{2\pi r} = \boxed{\frac{\mu_0 i}{2\pi r}}$$

b) $$d\Phi_B = B\,dA = B\,L\,dr = \boxed{\frac{\mu_0 i L\,dr}{2\pi r}}$$

c) $$\Phi_B = \int_{r=a}^{b} d\Phi_B = \frac{\mu_0 i L}{2\pi}\int \frac{dr}{r} = \boxed{\frac{\mu_0 i L}{2\pi}\ln\left(\frac{b}{a}\right)}$$

d) $$\varepsilon = -\frac{d\Phi_B}{dt} = \boxed{-\frac{\mu_0 L}{2\pi}\ln\left(\frac{b}{a}\right)\frac{di}{dt}}$$ counter clock wise

e) $$|\varepsilon| = \frac{\cancel{4\pi}^{2} \times 10^{-7}\,\frac{Tm}{A}\,(.24\,m)\,\ln\left(\frac{36}{12}\right)\,9.6\,\frac{A}{s}}{\cancel{2\pi}}$$

$$= 5.062 \times 10^{-7} \text{ volts}$$

**29.56 Terminal Speed.** A bar of length $L = 0.8$ m is free to slide without friction on horizontal rails, as shown in Fig. 29.40. There is a uniform magnetic field $B = 1.5$ T directed into the plane of the figure. At one end of the rails there is a battery with emf $\mathcal{E} = 12$ V and a switch. The bar has mass 0.90 kg and resistance 5.0 Ω, and all other resistance in the circuit can be ignored. The switch is closed at time $t = 0$. a) Sketch the speed of the bar as a function of time. b) Just after the switch is closed, what is the acceleration of the bar? c) What is the acceleration of the bar when its speed is 2.0 m/s? d) What is the terminal speed of the bar?

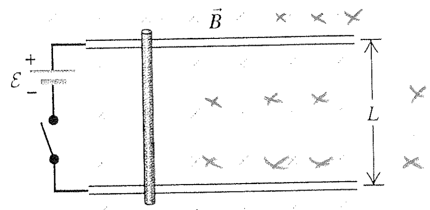
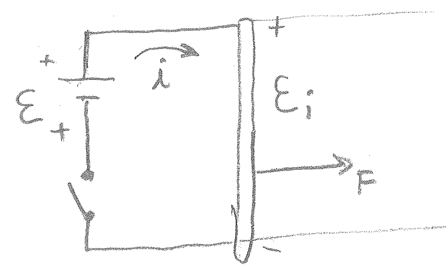



**Figure 29.40** Problem 29.56.

a)



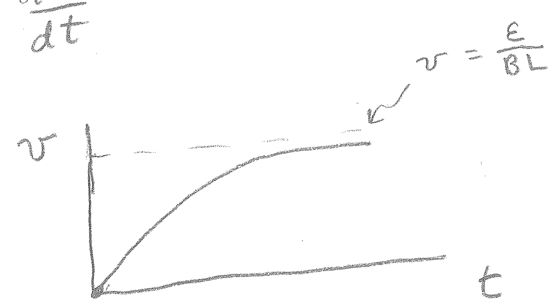

$\mathcal{E}_i = BvL$

induced

$$i = \frac{\mathcal{E} - \mathcal{E}_i}{R} = \frac{\mathcal{E} - (BvL)}{R}$$

$$F = iLB = \left(\frac{\mathcal{E} - BvL}{R}\right) LB = m\frac{dv}{dt}$$

$$\Rightarrow m\frac{dv}{dt} = \frac{\mathcal{E}}{R}LB - \frac{B^2L^2}{R}v$$




b) $t = 0$ $v = 0$

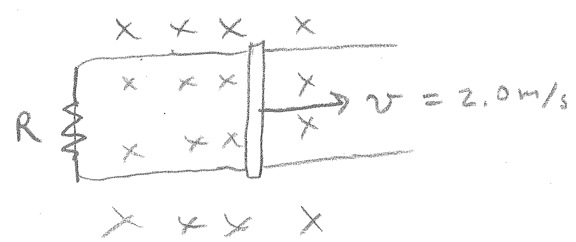
$$m\frac{dv}{dt} = \frac{\mathcal{E}}{R}LB \Rightarrow \frac{dv}{dt} = \frac{\mathcal{E}LB}{Rm} = \frac{(12\text{ V})(.8\text{ m})(1.5\text{ T})}{(5\,\Omega)(.9\text{ kg})}$$

$$= \boxed{3.2\ \text{m/s}^2}$$

(see other page)

**29.64 A Magnetic Exercise Machine.** You have designed a new type of exercise machine with an extremely simple mechanism (see Fig. 29.31). A vertical bar of silver (chosen for its low resistivity and because it makes the machine look cool) with length $L = 3.0$ m is free to move left or right without friction on silver rails. The entire apparatus is placed in a horizontal uniform magnetic field of strength 0.25 T. When you push the bar to the left or right, the bar's motion sets up a current in the circuit that includes the bar. The resistance of the bar and the rails can be neglected. The magnetic field exerts a force on the current-carrying bar, and this force opposes the bar's motion. The health benefit is from the exercise that you do in working against this force. a) Your design goal is that the person doing the exercise is to do work at the rate of 25 watts when moving the bar at a steady 2.0 m/s. What should be the resistance $R$? b) You decide you want to be able to vary the power required from the person, to adapt the machine to the person's strength and fitness. If the power is to be increased to 50 W by altering $R$ while leaving the other design parameters constant, should $R$ be increased or decreased? Calculate the value of $R$ for 50 W. c) When you start to construct a prototype machine, you find it is difficult to produce a 0.25-T magnetic field over such a large area. If you decrease the length of the bar to 0.20 m while leaving $B$, $v$, and $R$ the same as in part (a), what will be the power required of the person?

a)




$$\mathcal{E} = vBL \qquad \text{Power} = \frac{\mathcal{E}^2}{R} = \frac{(vBL)^2}{R}$$

$$\Rightarrow R = \frac{(vBL)^2}{\text{Power}} = \frac{(2\,m/s)^2(.25\,T)^2(3\,m)^2}{25\,W} = \boxed{0.09\,\Omega}$$

b) R should be decreased $\quad R' = R\,\frac{P}{P'} = 0.09\,\Omega\,\frac{25}{50}$

$$\Rightarrow R' = \boxed{0.045\,\Omega}$$

c) $\text{Power} = P = \frac{(vBL)^2}{R} \Rightarrow P \propto L^2 \Rightarrow P' = P\left(\frac{L'}{L}\right)^2$

$$\Rightarrow P' = 25\text{ watts}\left(\frac{.2}{3}\right)^2 = \boxed{0.11\text{ watts}}$$

29.56 c) $a = \frac{dv}{dt}$

$$\frac{dv}{dt} = \left(\frac{\mathcal{E} - Bv L}{mR}\right) LB = \left(\frac{(12\,V) - (1.5\,T)(2\,m/s)(.8\,m)}{(.9\,kg)(5\,\Omega)}\right)(.8\,m)(1.5\,T)$$

$$= \boxed{2.56\ m/s^2}$$

d)

$\frac{dv}{dt} = 0$ at $v_t$ terminal speed

$$\Rightarrow \quad \mathcal{E} - B v_t L = 0 \quad \Rightarrow \quad v = \frac{\mathcal{E}}{BL} = \frac{12\,V}{(1.5\,T)(.8\,m)}$$

$$= \boxed{10\ m/s}$$

**29.76** A metal bar with length $L$, mass $m$, and resistance $R$ is placed on frictionless, metal rails that are inclined at an angle $\phi$ above the horizontal. The rails have negligible resistance. A uniform magnetic field of magnitude $B$ is directed downward as shown in Fig. 29.49. The bar is released from rest and slides down the rails. a) Is the direction of the current induced in the bar from $a$ to $b$ or from $b$ to $a$? b) What is the terminal speed of the bar? c) What is the induced current in the bar when the terminal speed has been reached? d) After the terminal speed has been reached, at what rate is electrical energy being converted to thermal energy in the resistance of the bar? e) After the terminal speed has been reached, at what rate is work being done on the bar by gravity? Compare your answer to that in part (d).

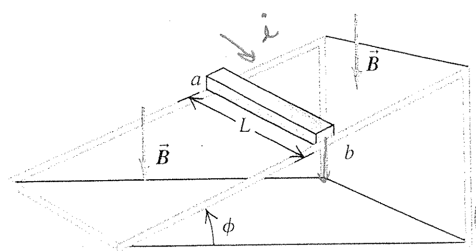
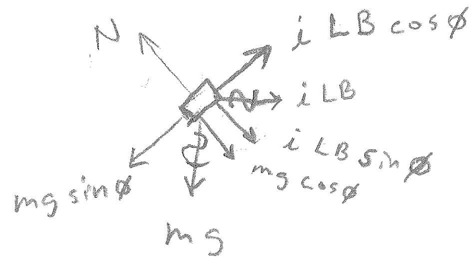



**Figure 29.49** Challenge Problem 29.76.

a) Lenz's Law

$$\boxed{a \text{ to } b}$$

b) FBD




(1) $\sum F_x = i L B \cos\phi - mg \sin\phi = m a_x$

$$i = \frac{\mathcal{E}}{R} = \frac{v B \cos\phi \, L}{R} \Rightarrow \frac{v B^2 L^2 \cos^2\phi}{R} - mg \sin\phi = m a_x$$

at $v = v_t$ $a_x = 0$ $\Rightarrow \frac{v_t (B L \cos\phi)^2}{R} = mg \sin\phi$

$$\Rightarrow \boxed{v_t = \frac{mg \sin\phi \quad R}{(BL \cos\phi)^2}}$$

c) from (1) $i_t L B \cos\phi = mg \sin\phi$ with $a_x = 0$

$$\Rightarrow i_t = \frac{mg \sin\phi}{L B \cos\phi} = \boxed{\frac{mg}{LB} \tan\phi}$$

d)

$$P = i^2 R = \boxed{\left(\frac{mg}{LB} \tan\phi\right)^2 R}$$

e) $P_{gravity} = (mg \sin\phi) v_t = \frac{(mg \sin\phi)^2}{(BL \cos\phi)^2} R$ = same as (d)