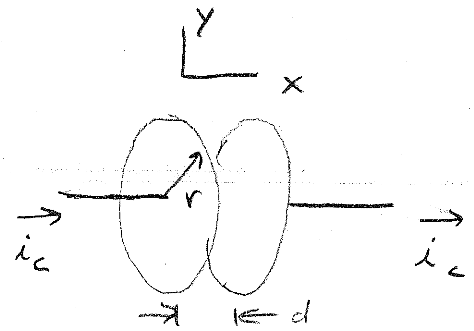
HW 20

29.35 A parallel-plate air-filled capacitor is being charged as in Fig. 29.22. The circular plates have radius 4.00 cm. and at a particular instant the conduction current in the wires is 0.280 A. a) What is the displacement current density j_D in the air space between the plates? b) What is the rate at which the electric field between the plates is changing? c) What is the induced magnetic field between the plates at a distance of 2.00 cm from the axis? d) at 1.00 cm from the axis?



$r = 0.04\,m$

a) $j_D \simeq \dfrac{i_c}{A} = \dfrac{.28A}{\pi(0.04m)^2}$ $\qquad C = \dfrac{\epsilon_0 A}{d}$

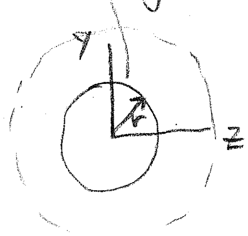
$\cong \boxed{55.7\ \dfrac{A}{m^2}}$

b) $C = \dfrac{Q}{V} = V = \dfrac{Q}{C} \Rightarrow E = \dfrac{V}{d} = \dfrac{Q}{Cd}$

$\Rightarrow \left|\dfrac{dE}{dt}\right| = \dfrac{i}{Cd} = \dfrac{i}{\left(\frac{\epsilon_0 A}{\cancel{d}}\right)\cancel{d}} = \dfrac{i}{\epsilon_0 A} = \dfrac{j_D}{\epsilon_0} = \dfrac{55.70423\ \frac{A}{m^2}}{8.85\times10^{-12}\ \frac{C^2}{Nm^2}}$

$= \boxed{6.294\times10^{12}\ \dfrac{N}{Cs}}$

c) $\oint \vec{B}\cdot d\vec{l} = \mu_0\epsilon_0\dfrac{d\Phi_E}{dt} \Rightarrow B(2\pi r) = \mu_0\epsilon_0(\pi r^2)\dfrac{\partial E}{\partial t}$



$\Rightarrow B = \dfrac{\mu_0\epsilon_0}{2} r\dfrac{\partial E}{\partial t} = \dfrac{\mu_0\cancel{\epsilon_0}}{2} r\dfrac{j_D}{\cancel{\epsilon_0}} = \dfrac{\mu_0 r j_D}{2}$

$= \dfrac{(4\pi\times10^{-7}\ \frac{Tm}{A})(0.02m)(55.70423\ \frac{A}{m^2})}{2} = \boxed{7.00\times10^{-7}\ T}$

d) $B' = B\dfrac{r'}{r} = (7.\times10^{-7}T)\left(\dfrac{1}{2}\right) = \boxed{3.5\times10^{-7}\ T}$

30.1 Two coils have mutual inductance $M = 3.25 \times 10^{-4}$ H. The current i_1 in the first coil increases at a uniform rate of 830 A/s. a) What is the magnitude of the induced emf in the second coil? Is it constant? b) Suppose that the current described is in the second coil rather than the first. What is the magnitude of the induced emf in the first coil?

a) $\mathcal{E}_2 = -M \frac{di_1}{dt} = -(3.25 \times 10^{-4} \cancel{H})\, 830 \frac{\cancel{A}}{\cancel{s}} \frac{V\cancel{s}/\cancel{A}}{\cancel{H}}$

$\Rightarrow |\mathcal{E}_2| = \boxed{2.698 \times 10^{-1} \text{ Volts}}$

b) $\mathcal{E}_1 = -M \frac{di_2}{dt} = \underline{\text{same}}$ as (a)

30.3 Two toroidal solenoids are wound around the same form so that the magnetic field of one passes through the turns of the other. Solenoid 1 has 700 turns and solenoid 2 has 400 turns. When the current in solenoid 1 is 6.52 A, the average flux through each turn of solenoid 2 is 0.0320 Wb. a) What is the mutual inductance of the pair of solenoids? b) When the current in solenoid 2 is 2.54 A, what is the average flux through each turn of solenoid 1?

a)

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{(400)(0.032)\,\text{Wb}}{6.52\ \text{A}} = \boxed{1.96\ \text{H}}$$

b)

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2} \Rightarrow \Phi_{B1} = \frac{M i_2}{N_1} = \frac{(1.963\ \text{H})(2.54\text{A})}{(700)}$$

$$\Rightarrow \Phi_{B1} = \boxed{0.00712\ \text{Wb}}$$

30.11 At the instant when the current in an inductor is increasing at a rate of 0.0640 A/s, the magnitude of the self-induced emf is 0.0160 V. a) What is the inductance of the inductor? b) If the inductor is a solenoid with 400 turns, what is the average magnetic flux through each turn when the current is 0.720 A?

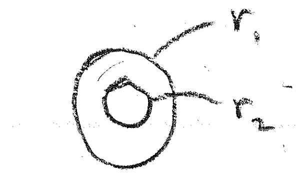
a) $$\mathcal{E} = -L\frac{di}{dt} \Rightarrow L = \left|\frac{\mathcal{E}}{\frac{di}{dt}}\right| = \frac{.016\,V}{.064\,A/s}$$

$$= \boxed{0.25\,H}$$

b) $$L = \frac{N\Phi_B}{i} \Rightarrow \Phi_B = \frac{iL}{N} = \frac{(.72\,A)(0.25\,H)}{400}$$

$$= \boxed{4.5 \times 10^{-4}\ Wb}$$

30.46 A solenoid has length l_1, radius r_1, and number of turns N_1. A second, smaller solenoid with length l_2, radius r_2 $(r_2 < r_1)$, and number of turns N_2 is placed at the center of the first solenoid, such that their axes coincide. Assume that the magnetic field of the first solenoid, at the location of the second, is uniform and has a magnitude given by the equation derived in Example 28.10 (Section 28.7). a) What is the mutual inductance of the pair of solenoids? b) If the current in the large solenoid is increasing at the rate di_1/dt, what is the magnitude of the emf induced in the small solenoid? c) If the current in the smaller solenoid is increasing at the rate di_2/dt, what is the magnitude of the emf induced in the larger solenoid?



a) We are told How to calculate the $\vec{B}$-field from the first solenoid, so we must get the mutual inductance from that $\vec{B}$-field.

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2}{i_1} B_1 A_2 = \frac{N_2}{i}\left(\mu_0 \frac{N_1}{l_1} i_1\right)\left(\pi r_2^2\right)$$

$$= \boxed{\frac{\mu_0 N_1 N_2 \pi r_2^2}{l_1}}$$

b)
$$|\mathcal{E}_2| = M \frac{di_1}{dt} = \boxed{\frac{\mu_0 N_1 N_2 \pi r_2^2}{l_1} \frac{di_1}{dt}}$$

c)
$$|\mathcal{E}_1| = M \frac{di_2}{dt} = \boxed{\frac{\mu_0 N_1 N_2 \pi r_2^2}{l_1} \frac{di_2}{dt}}$$

30.48 A coil has 400 turns and self-inductance 3.50 mH. The current in the coil varies with time according to $i = (680 \text{ mA}) \cos[\pi t/(0.0250 \text{ s})]$. a) What is the maximum emf induced in the coil? b) What is the maximum average flux through each turn of the coil? c) At $t = 0.0180$ s, what is the magnitude of the induced emf?

a) $\mathcal{E} = -L\dfrac{di}{dt}$ $\quad |\mathcal{E}_{max}| = \left|L\dfrac{di}{dt}\right|_{max}$

$$L\frac{di}{dt} = L\frac{d}{dt} i_0 \cos(\omega t) = -L i_0 \omega \sin(\omega t)$$

$$\Rightarrow \mathcal{E}_{max} = L i_0 \omega = (3.5 \text{ mH})(680 \text{ mA})\left(\frac{\pi}{0.025 \text{ s}}\right)$$

$$= \boxed{0.2991 \text{ volts}}$$

b) $L = \dfrac{N \Phi_B}{i} \Rightarrow \Phi_B = \dfrac{iL}{N}$

$$\Phi_{B\,max} = \frac{i_0 L}{N} = \frac{(680 \text{ mA})(3.50 \text{ mH})}{400} = \boxed{5.95 \ \mu\text{Wb}}$$

c) $|\mathcal{E}| = \mathcal{E}_{max}|\cos(\omega t)| = \mathcal{E}_{max}\left|\sin\left(\pi \dfrac{.018 \text{ s}}{.025 \text{ s}}\right)\right|$

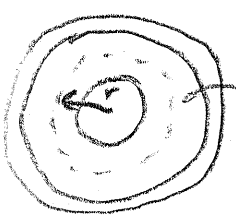
$$= \boxed{0.230 \text{ volts}}$$

30.50 A Coaxial Cable. A small solid conductor with radius a is supported by insulating, nonmagnetic disks on the axis of a thin-walled tube with inner radius b. The inner and outer conductors carry equal currents i in opposite directions. a) Use Ampere's law to find the magnetic field at any point in the volume between the conductors. b) Write the expression for the flux $d\Phi_B$ through a narrow strip of length l parallel to the axis, of width dr, at a distance r from the axis of the cable and lying in a plane containing the axis. c) Integrate your expression from part (b) over the volume between the two conductors to find the total flux produced by a current i in the central conductor. d) Show that the inductance of a length l of the cable is

$$L = l\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

e) Use Eq. (30.9) to calculate the energy stored in the magnetic field for a length l of the cable.

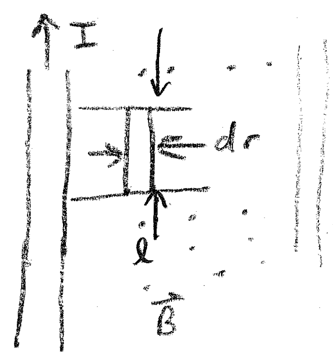
a)



$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r) = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \qquad a < r < b}$$

b)



$$d\Phi_B = \left(\frac{\mu_0 I}{2\pi r}\right) l\, dr$$

c) $$\Phi_B = \frac{\mu_0 I l}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I l}{2\pi}\ln\left(\frac{b}{a}\right)$$

d) $$L = \frac{N\Phi_B}{I} = \frac{N\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right), \quad N=1 \quad = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right)$$

e) $$U = \frac{1}{2}LI^2 = \frac{\mu_0 l I^2}{4\pi}\ln\left(\frac{b}{a}\right)$$

30.53 Uniform electric and magnetic fields $\vec{E}$ and $\vec{B}$ occupy the same region of free space. If $E = 650$ V/m, what is B if the energy densities in the electric and magnetic fields are equal?

$$u_B = \frac{B^2}{2\mu_0} \qquad u_E = \frac{1}{2}\epsilon_0 E^2$$

$$u_B = u_E \implies B^2 = \mu_0 \epsilon_0 E^2$$

$$\implies B = \sqrt{\mu_0 \epsilon_0}\, E = \sqrt{\left(4\pi \times 10^{-7} \frac{Tm}{A}\right) 8.85 \times 10^{-12} \frac{C^2}{Nm^2}}\; 650 \frac{V}{m}$$

$$\cong \boxed{2.17 \times 10^{-6}\ T}$$