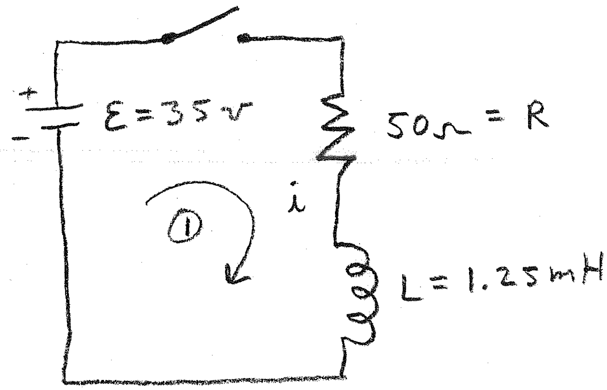
HW21

30.21 A 35.0-V battery with negligible internal resistance, a 50.0-Ω resistor, and a 1.25-mH inductor with negligible resistance are all connected in series with an open switch. The switch is suddenly closed. a) How long after closing the switch will the current through the inductor reach one-half of its maximum value? b) How long after closing the switch will the energy stored in the inductor reach one-half of its maximum value?



KVL 1 $\Rightarrow$ $\mathcal{E} - iR - L\frac{di}{dt} = 0$

Voltage Loop Rule

$$\Rightarrow \frac{di}{dt} = \frac{\mathcal{E}}{L} - \frac{R}{L}i \Rightarrow \frac{\left(\frac{di}{dt}\right)dt}{-\frac{R}{L}i + \frac{\mathcal{E}}{L}} = dt$$

$$\int_{i=0}^{i} \frac{di'\,L/R}{-i' + \frac{\mathcal{E}}{R}} = \int_0^t dt' \Rightarrow -\frac{L}{R}\ln\left(\frac{\frac{\mathcal{E}}{R} - i}{\frac{\mathcal{E}}{R}}\right) = t$$

$$\frac{\mathcal{E}}{R} - i = \frac{\mathcal{E}}{R}e^{-\frac{tR}{L}} \Rightarrow i = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)$$

$$\Rightarrow i_{max} = \frac{\mathcal{E}}{R} \qquad t_{1/2} = -\frac{L}{R}\ln\left(\frac{1}{2}\right) = -\frac{1.25\,mH}{50\Omega}\ln\left(\frac{1}{2}\right) \cong \boxed{17.3\,\mu s}$$

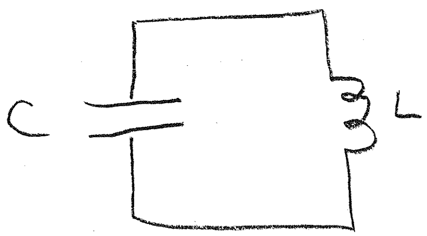
b) $U = \frac{1}{2}Li^2 \qquad U_{max} = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2 = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2\left(1 - e^{-tR/L}\right)^2$

$$U = \frac{U_{max}}{2} \Rightarrow \left(1 - e^{-tR/L}\right)^2 = \frac{1}{2} \Rightarrow 1 - e^{-tR/L} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow e^{-tR/L} = \left(1 - \frac{1}{\sqrt{2}}\right) \Rightarrow -\frac{tR}{L} = \ln\left(1 - \frac{1}{\sqrt{2}}\right) \Rightarrow t = -\frac{L}{R}\ln\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow t_{\frac{1}{2}\,Power} = -\frac{1.25\,mH}{50\Omega}\ln\left(1 - \frac{1}{\sqrt{2}}\right) \cong \boxed{30.7\,\mu s}$$

30.31 When a voltmeter is placed across the plates of a capacitor, it reads 4.29 mV when the plates carry a charge of magnitude 150 nC. If the capacitor is now charged to 45.0 V and connected across a coil of negligible resistance, you observe that the current in the circuit oscillates with a period of 911 μs. Find the capacitance of the capacitor and the inductance of the coil.



$$C = \frac{Q}{V} = \frac{150\, \cancel{n}C}{4.29\, \cancel{m}V} \left(\frac{10^{-9}\, \cancel{n}}{\cancel{m}\, 10^{-3}} \right)$$

$$= \boxed{34.96\ \mu C}$$

$$\omega = \sqrt{\frac{1}{LC}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$

$$\Rightarrow LC = \left(\frac{T}{2\pi}\right)^2 \Rightarrow L = \frac{T^2}{(2\pi)^2}\frac{1}{C} = \frac{(911\ \mu s)^2}{(2\pi)^2}\frac{1}{(34.96\ \mu C)}$$

$$\Rightarrow L = \boxed{601\ \mu H}$$

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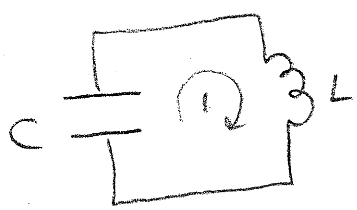
digression

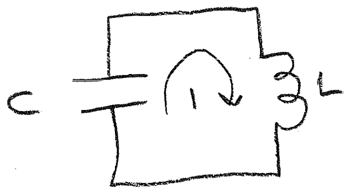


$$KVL① \Rightarrow -\frac{q}{C} - L\frac{di}{dt} = 0$$

$$\Rightarrow \frac{di}{dt} = -\frac{C}{L}q \Rightarrow \frac{d^2 i}{dt^2} = -\underbrace{\left(\frac{C}{L}\right)}_{\omega^2} i \Rightarrow \underline{SHM}$$

Simple Harmonic Motion

30.32 In an L-C circuit, $L = 85.0$ mH and $C = 3.20\ \mu$F. During the oscillations the maximum current in the inductor is 0.850 mA. a) What is the maximum charge on the capacitor? b) What is the magnitude of the charge on the capacitor at an instant when the current in the inductor has magnitude 0.500 mA?



$$\text{KVL}① \Rightarrow -\frac{q}{C} - L\frac{di}{dt} = 0$$

$$\Rightarrow q = -LC\frac{di}{dt} \qquad |q_{max}| = |LC|\left|\frac{di}{dt}\right|_{max}$$

$$i = i_{max}\sin\left(\frac{t}{\sqrt{LC}} + \phi_0\right) \qquad \frac{di}{dt} = \frac{i_{max}}{\sqrt{LC}}\cos\left(\frac{t}{\sqrt{LC}} + \phi\right)$$

$$\Rightarrow |q_{max}| = LC\frac{i_{max}}{\sqrt{LC}}\left|\cos\left(\frac{t}{\sqrt{LC}} + \phi\right)\right|_{max} = \sqrt{LC}\, i_{max}$$

$$= \sqrt{85\,\cancel{m}H\,(3.2\,\cancel{\mu}F)(10^{-3})(10^{-6})}\ .85\text{ mA}$$

$$= \boxed{4.433 \times 10^{-7}\ C}$$

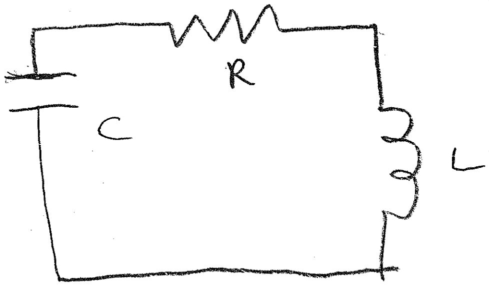
$$q \overset{?}{=} LC\frac{di}{dt}$$

SHM $\Rightarrow$ $$\frac{1}{\cancel{2}}\omega^2 i_{max}^2 = \frac{1}{\cancel{2}}\omega^2 i^2 + \frac{1}{\cancel{2}}\left(\frac{di}{dt}\right)^2 \Rightarrow \frac{di}{dt} = \omega\sqrt{i_{max}^2 - i^2}$$

$$\Rightarrow q = LC\,\omega\sqrt{i_{max}^2 - i^2} = \sqrt{LC}\sqrt{i_{max}^2 - i^2} = q_{max}\sqrt{1 - \left(\frac{i}{i_{max}}\right)^2}$$

$$= (4.433 \times 10^{-7}\ C)\sqrt{1 - \left(\frac{.5}{.85}\right)^2} = \boxed{3.585 \times 10^{-7}\ C}$$

30.42 An *L-R-C* circuit has $L = 0.450$ H, $C = 2.50 \times 10^{-5}$ F, and resistance R. a) What is the angular frequency of the circuit when $R = 0$? b) What value must R have to give a 5.0% decrease in angular frequency compared to the value calculated in part (a)?



a)

$$\omega = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{.45\,H\,(2.5 \times 10^{-5} F)}}$$

$$= \boxed{298.142\ \frac{rad}{s}}$$

b)

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$\frac{\omega - \omega'}{\omega} = 0.05 = \frac{\sqrt{\frac{1}{LC}} - \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}{\sqrt{\frac{1}{LC}}}$$

$$= \frac{1 - \sqrt{1 - \frac{R^2 LC}{4L^2}}}{1}$$

$$\Rightarrow \sqrt{1 - \frac{R^2 C}{4L}} = 1 - 0.05$$

$$\Rightarrow 1 - \frac{R^2 C}{4L} = (.95)^2 \Rightarrow \frac{R^2 C}{4L} = 1 - (.95)^2$$

$$\Rightarrow R^2 = \frac{4L}{C}\left[1 - (.95)^2\right] = \frac{4\,(.45\,H)}{(2.5 \times 10^{-5} F)}\left[1 - (.95)^2\right]$$

$$\Rightarrow R \cong \boxed{83.8\ \Omega}$$

30.64 In the circuit shown in Fig. 30.23, find the reading in each ammeter and voltmeter at the following times a) just after switch S is closed; b) after S has been closed a very long time.

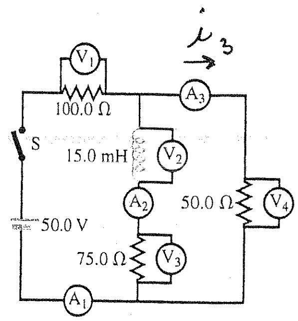
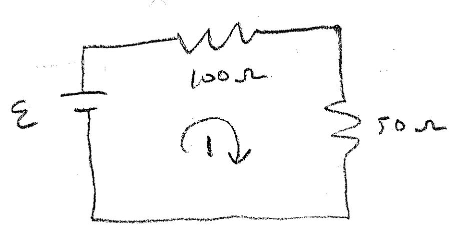


Figure 30.23 Problem 30.64.

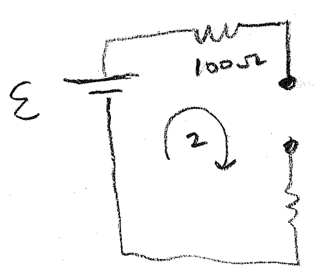
a) When the switch is closed, at $t=0$, the inductor will act like a open circuit for an instant.



$$KVL \; ① \Rightarrow i_3 = \frac{50\,v}{150\,\Omega}$$

$$\Rightarrow i_3 = \frac{1}{3}\,A$$

$$V_1 = i_3 R_1 = \left(\tfrac{1}{3}A\right) 100\,\Omega = 33.\bar{3}\,v$$



$$V_4 = i_3 R_4 = \left(\tfrac{1}{3}A\right) 50\,\Omega = 16.\bar{6}\,v$$

$$V_3 = 0 \qquad i_1 = i_3 = \frac{1}{3}\,A$$

$$i_2 = 0$$

$$KVL \; ② \Rightarrow \varepsilon - i_3 R_3 - V_2 = 0$$

$$\Rightarrow V_2 = \varepsilon - i_3 R_3 = 50\,v - 33.\bar{3}\,v = 16.\bar{6}\,v$$

b) after $t \to \infty$ the inductor is like a short.

$$i_1 = \frac{50\,v}{100\,\Omega + \left(\frac{75(50)}{75+50}\right)\Omega} = 0.3846\,A \qquad V_1 = i_1 R_1 = 38.46\,v$$

$$i_2 = \frac{V_3}{R_3} = \boxed{0.154\,A}$$

$$V_4 = \varepsilon - V_1 = 50\,v - 38.4615\,v = 11.54\,v = V_3 \Rightarrow$$

$$i_3 = i_1 - i_2 = 0.231\,A$$

30.68 In the circuit shown in Fig. 30.27, the capacitor is originally uncharged. The switch starts in the open position and is then flipped to position 1 for 0.500 s. It is then flipped to position 2 and left there. a) If the resistance r is very small, what is the upper limit for the amount of charge the capacitor could receive? b) Even if r is very small, how much electrical energy will be dissipated in it? c) Sketch a graph showing the reading of the ammeter as a function of time after the switch is in position 2, assuming that r is small.

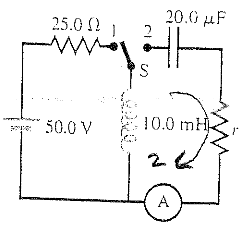
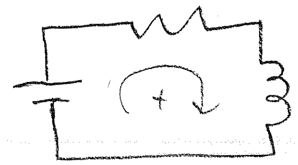


Figure 30.27 Problem 30.68.



KVL ① $\Rightarrow \mathcal{E} - iR - L\frac{di}{dt} = 0$

$\Rightarrow \frac{di}{dt} = \frac{\mathcal{E}}{L} - i\frac{R}{L}$

$\Rightarrow i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)$

a)

$$i\left(\tfrac{1}{2}s\right) = \frac{50V}{25\Omega}\left(1 - \underbrace{e^{-\frac{(.5)s \; 25\Omega}{10\,mH}}}_{\sim 0}\right)$$

$C = \frac{Q}{V}$

$$\cong \frac{50V}{25\Omega}$$

$$U_L = \frac{1}{2}LI^2 = U_c = \frac{1}{2}\frac{Q^2}{C}$$

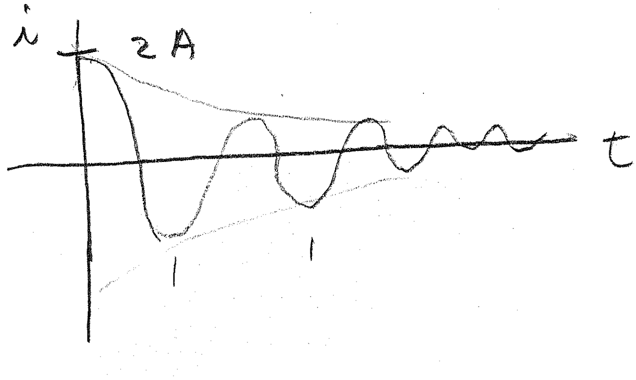
$$\Rightarrow \frac{Q^2}{C} = LI^2 \Rightarrow Q^2 = (LC)I^2 \Rightarrow Q = \sqrt{LC}\,I$$

$$\Rightarrow Q = \sqrt{10\times10^{-3}H \;\; 20\times10^{-6}F}\;\frac{50V}{25\Omega} = \boxed{8.944\times10^{-4}\,C}$$

b) ALL energy with go through the resistor r.

$$U = U_L = \frac{1}{2}LI^2 = \frac{1}{2}(10\,mH)\left(\frac{50V}{25\Omega}\right)^2 = 20\,mJ = \boxed{0.02\,J}$$

c)



$t = 0$ [inductor] $=$ —o o—

30.69 In the circuit shown in Fig. 30.28, $\mathcal{E} = 60.0$ V, $R_1 = 40.0\ \Omega$, $R_2 = 25.0\ \Omega$, and $L = 0.300$ H. Switch S is closed at $t = 0$. Just after the switch is closed, a) What is the potential difference v_{ab} across the resistor R_1? b) Which point, a or b, is at a higher potential? c) What is the potential difference v_{cd} across the inductor L? d) Which point, c or d, is at a higher potential? The switch is left closed a long time and then opened. Just after the switch is opened, e) What is the potential difference v_{ab} across the resistor R_1? f) Which point, a or b, is at a higher potential? g) What is the potential difference v_{cd} across the inductor L? h) Which point, c or d, is at a higher potential?

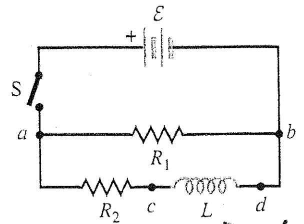


Figure 30.28 Problems 30.69, 30.70, and 30.75.

a) 60 volts

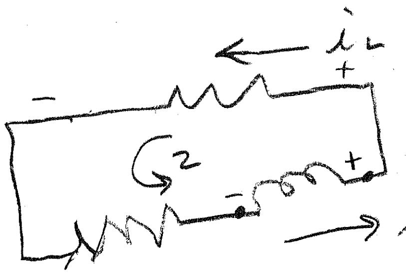
b) $\boxed{a}$

c) $\mathcal{E} = 60$ volts

d) $\boxed{c}$

e) $i_L(t=\infty) = \frac{\mathcal{E}}{R_2} = \frac{60\text{ V}}{25\ \Omega} = 2.4$ A

The current will be $i_L(t=\infty) = 2.4$ A



$\therefore\ V_{ab} = R_1 i_L = 40\ \Omega\ (2.4\text{ A}) = 96$ volts

f) $\boxed{b}$ from the direction of the current

g) KVL ② $\Rightarrow -i_L R_1 - i_L R_2 - V_L = 0$

$\Rightarrow -V_L = i_L R_1 + i_L R_2 = (2.4\text{A})(40\ \Omega + 25\ \Omega)$

$= \boxed{156 \text{ volts}}$

h) $\boxed{d}$

30.74 In the circuit shown in Fig. 30.31, neither the battery nor the inductors have any appreciable resistance, the capacitors are initially uncharged, and the switch S has been in position 1 for a very long time. Review the results of Problem 30.49. a) What is the current in the circuit? b) The switch is now suddenly flipped to position 2. Find the maximum charge that each capacitor will receive, and how much time after the switch is flipped it will take them to acquire this charge.

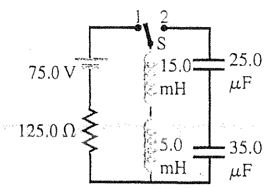
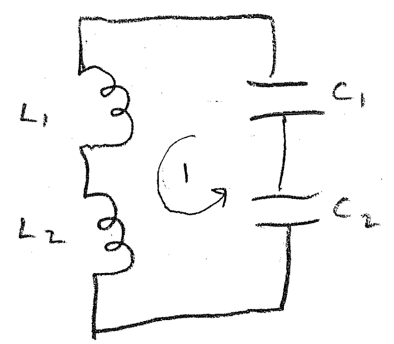


Figure 30.31 Problem 30.74.

a) The inductors act like a short.

$$\Rightarrow I = \frac{\mathcal{E}}{R} = \frac{75\,V}{125\,\Omega} = \boxed{0.6\ A}$$

b)



$$i(t=0) = .6\ A$$

$$\text{KVL } ① \Rightarrow -L_1\frac{di}{dt} - L_2\frac{di}{dt} - \frac{q}{C_1} - \frac{q}{C_2} = 0$$

$$\Rightarrow (L_1+L_2)\frac{di}{dt} = -\left(\frac{C_1+C_2}{C_1C_2}\right)q$$

$$\Rightarrow \frac{d^2q}{dt^2} = \frac{-C_1+C_2}{C_1C_2(L_1+L_2)}\,q \quad \Rightarrow \text{Simple Harmonic Motion}$$

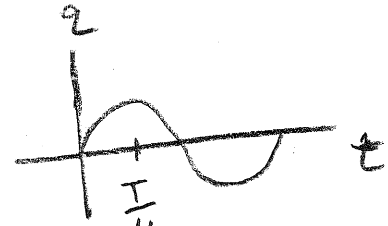
$$U_L = \frac{1}{2}L_{eq}\,i_{max}^2, \quad U_c = \frac{1}{2}\frac{Q^2}{C_{eq}}$$

Energy with switch from Capacitors to Inductors

$$U_L = U_c \Rightarrow Q^2 = C_{eq}L_{eq}\,i_{max}^2$$

$$Q = \sqrt{\frac{25(35)}{25+35}\times10^{-6}F\,(15+5)\times10^{-3}H}\ \ 0.6\,A$$

$$= \boxed{3.24037\times10^{-4}\ C}$$



Time to charge will be $\frac{1}{4}$ period of oscillation

$$= \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2}\sqrt{LC} = \boxed{8.48\times10^{-4}\ s}$$

30.77 Consider the circuit shown in Fig. 30.32. Switch S is closed at time $t = 0$, causing a current i_1 through the inductive branch and a current i_2 through the capacitive branch. The initial charge on the capacitor is zero, and the charge at time t is q_2. a) Derive expressions for i_1, i_2, and q_2 as functions of time. Express your answers in terms of $\mathcal{E}$, L, C, R_1, R_2, and t. For the remainder of the problem let the circuit elements have the following values: $\mathcal{E} = 48$ V, $L = 8.0$ H, $C = 20\ \mu$F, $R_1 = 25\ \Omega$, and $R_2 = 5000\ \Omega$. b) What is the initial current through the inductive branch? What is the initial current through the capacitive branch? c) What are the currents through the inductive and capacitive branches a long time after the switch has been closed? How long is a "long time"? Explain. d) At what time t_1 (accurate to two significant figures) will the currents i_1 and i_2 be equal? (*Hint:* You might consider using series expansions for the exponentials.) e) For the conditions given in part (d), determine i_1. f) The total current through the battery is $i = i_1 + i_2$. At what time t_2 (accurate to two significant figures) will i equal one-half of its final value? (*Hint:* The numerical work is greatly simplified if one makes suitable approximations. A sketch of i_1 and i_2 versus t may help you decide what approximations are valid.)

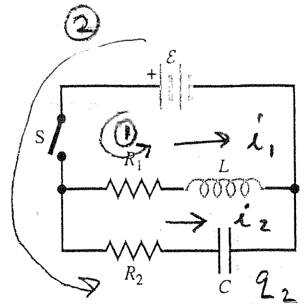


Figure 30.32 Challenge Problem 30.77.

$$\text{KVL } (1) \Rightarrow \mathcal{E} - i_1 R_1 - L\frac{di_1}{dt} = 0$$

$$\text{KVL } (2) \Rightarrow \mathcal{E} - i_2 R_2 - \frac{q_2}{C} = 0$$

The two loops act independent of each other, because $\mathcal{E}$ is constant,

$$i_1(t) = \frac{\mathcal{E}}{R_1}\left(1 - e^{-\frac{tR_1}{L}}\right)$$

$$i_2(t) = \frac{\mathcal{E}}{R_2} e^{-t/R_2C}$$

$$q_2(t) = \mathcal{E}C\left(1 - e^{-t/R_2C}\right)$$

b) $i_1(t=0) = 0$ $\qquad i_2(t=0) = \frac{\mathcal{E}}{R_2} = \frac{48\text{ V}}{5000\ \Omega} = \underline{9.6\text{ mA}}$

c) $i_1(t=\infty) = \frac{\mathcal{E}}{R_1} = \frac{48\text{ V}}{25\ \Omega} = \underline{1.92\text{ A}}$ $\qquad i_2(t=\infty) = 0$

$\tau_L = \frac{L}{R_1} = \frac{8\text{ H}}{25\ \Omega} = 0.32\text{ s}$ $\qquad \tau_c = R_2 C = (5000\ \Omega)(20\times10^{-6}\text{ F})$
$= 0.1\text{ sec}$

time >> 0.32 s is a long time

d) $e^{-x} \sim 1 - x$

$$i_1(t_1) = i_2(t_1) \Rightarrow \frac{1 - e^{-\frac{t_1R_1}{L}}}{R_1} = \frac{e^{-t_1/R_2C}}{R_2}$$

$$\Rightarrow \left[\cancel{1} - \left(\cancel{1} - \frac{tR_1}{L}\right)\right]R_2 \cong R_1\left(1 - t/R_2C\right) \Rightarrow \frac{t_1 \cancel{R_1} R_2}{\cancel{R_1} L} = 1 - t_1/R_2C$$

$$\Rightarrow t_1\left(\frac{R_2}{L} + \frac{1}{R_2C}\right) = 1 \Rightarrow t_1 \simeq \frac{1}{\left(\frac{R_2}{L} + \frac{1}{R_2C}\right)} = \frac{1}{\left(\frac{5000\ \Omega}{8\text{ H}}\right) + \frac{1}{5000\ \Omega(20\times10^{-6})}}$$

$\Rightarrow t_1 = \boxed{1.599744\text{ ms}}$ this is small compared to $\frac{L}{R_1}$ and R_2C so it worked

$t_1 = .001599744\,s$

e) $i_1(t_1) = 1.92\,A\left(1 - e^{-t_1/.32\,s}\right)$

$= \boxed{9.57 \times 10^{-3}\,A}$

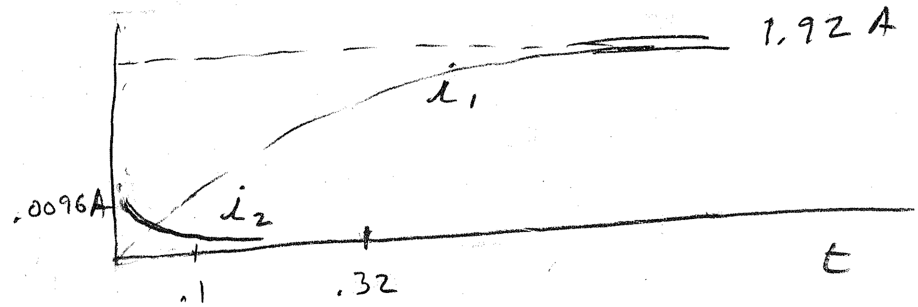
$i_2(t_1) = \frac{\mathcal{E}}{R_2} e^{-t_1/R_2C} = 9.6\,mA\,e^{-\frac{0.0015997}{0.1}}$

$\simeq 9.5 \times 10^{-3}\,A$ ✓

f) $i(t=\infty) = \frac{\mathcal{E}}{R_1}$

$i(t_2) = \frac{\mathcal{E}}{2R_1} = \frac{\mathcal{E}}{R_1}\left(1 - e^{-t_2 \frac{R_1}{L}}\right) + \frac{\mathcal{E}}{R_2} e^{-t_2/R_2C}$

↑
small



$i(t_2) \simeq \frac{\mathcal{E}}{R_1}\left(1 - e^{-t_2 \frac{R_1}{L}}\right) = \frac{\mathcal{E}}{2R_1}$

$\Rightarrow e^{-t_2 \frac{R_1}{L}} = \frac{1}{2} \Rightarrow \ln 2 = \frac{t_2 R_1}{L}$

$\Rightarrow t_2 = \frac{L}{R} \ln 2 = \frac{8H}{25\Omega} \ln 2 \cong \boxed{.222\,s}$

1

31.1 The voltage across the terminals of an ac power supply varies with time according to Eq. (31.1). The voltage amplitude is $V = 45.0$ V. What is a) the root-mean-square potential difference V_{rms}? b) the average potential difference V_{av} between the two terminals of the power supply?

Eq (31.1) $v = V\cos\omega t$

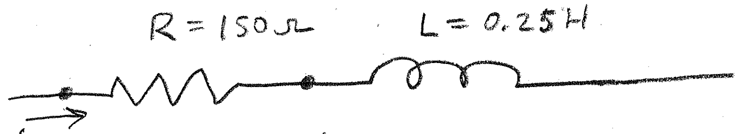
a)

$$V_{rms} = \sqrt{\frac{1}{\left(\frac{2\pi}{\omega}\right)}\int_0^{t'=\frac{2\pi}{\omega}} V^2\cos^2\omega t\ dt} = \frac{V}{\sqrt{2}} = \frac{45\ V}{\sqrt{2}} = \boxed{31.82\ V}$$

b)

$$V_{ave} = \frac{1}{\left(\frac{2\pi}{\omega}\right)}\int_0^{t'=\frac{2\pi}{\omega}} V\cos\omega t\ dt = \boxed{0}$$

31.9 A 150-Ω resistor is connected in series with a 0.250-H inductor. The voltage across the resistor is $v_R = (3.80\text{ V})\cos[(720\text{ rad/s})t]$. a) Derive an expression for the circuit current. b) Determine the inductive reactance of the inductor. c) Derive an expression for the voltage v_L across the inductor.



$v_R = V\cos\omega t$

a)

$$i_R = \frac{v_R}{R} = \frac{V}{R}\cos\omega t = \frac{3.8\text{ V}}{150\,\Omega}\cos\omega t$$

$$= \boxed{0.025\bar{3}\text{ A}\cos\left(720\,\tfrac{\text{rad}}{\text{s}}\,t\right)}$$

b) $X_L = \omega L = \left(720\,\frac{\text{rad}}{\text{s}}\right)(0.25\text{ H}) = \boxed{180\,\Omega}$ $\angle 90°$ Lead

v_L Leads i_L

c) $v_L = I X_L = \left(\dfrac{3.8\text{V}}{150\,\Omega}\right)(180\,\Omega)\cos(\omega t + 90°)$

$$= \boxed{-4.58\text{V}\sin\left(720\,\tfrac{\text{rad}}{\text{s}}\,t\right)}$$

or

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(\frac{V}{R}\cos\omega t\right) = \frac{LV}{R}\omega(-\sin\omega t)$$

$$= -4.58\text{ V}\sin\left(720\,\tfrac{\text{rad}}{\text{s}}\,t\right)$$