31.18, 37, 38, 46, 52

31.18 31.17

d) $v_R = IR = (0.04992\ A)(200\Omega) = \boxed{9.98V}$

$v_L = IX_L = (0.04992\ A)\left(\frac{250}{s}\right)(.4H) = \boxed{4.99V \angle 90^\circ \text{ Lead}}$

$v_c = IX_c = \dfrac{(0.04992\ A)}{\left(\frac{250}{s}\right)(6\times10^{-6}F)} = \boxed{33.28\ V \angle 90^\circ \text{ Lag}}$

e) Not in the same phase

31.18 from 31.17

a) $i = I\cos\omega t \qquad v = v_R(t) + v_L(t) + v_c(t)$

$v(t) = 30V\cos\left(\frac{250}{s}t - 70.56^\circ\right)$

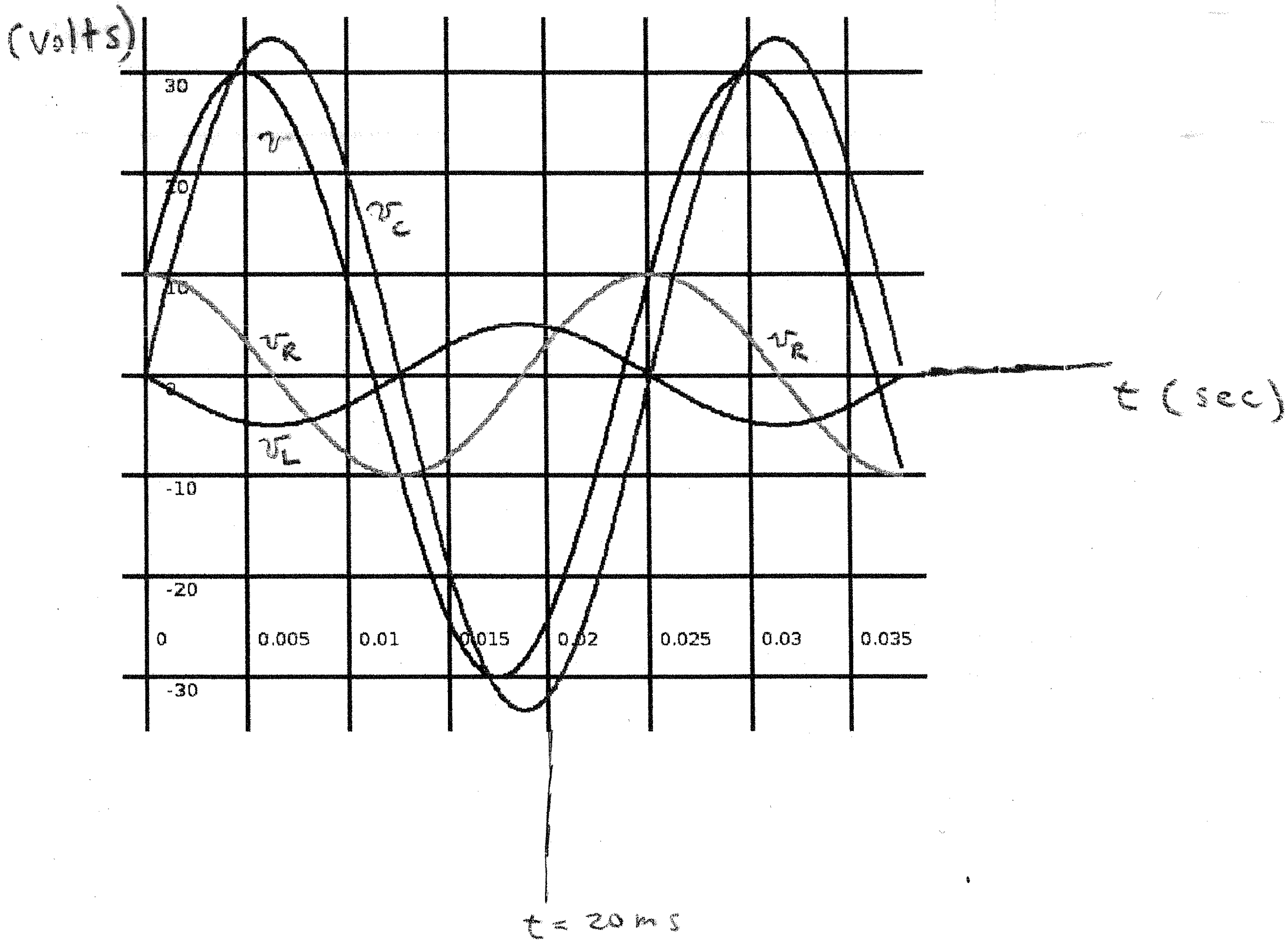
$v_R(t) = 9.98V\cos\left(\frac{250}{s}t\right)$

$v_L = 4.99V\cos\left(\frac{250}{s}t + 90^\circ\right)$

$v_c = 33.28V\cos\left(\frac{250}{s}t - 90^\circ\right)$

See over for plot

31.18 (continued)



$t = 20\,ms = 0.02\,s$

b) $v = -24.29\,V$, $v_R = 2.83\,V$ $v_L = 4.785\,V$, $v_C = -31.91$

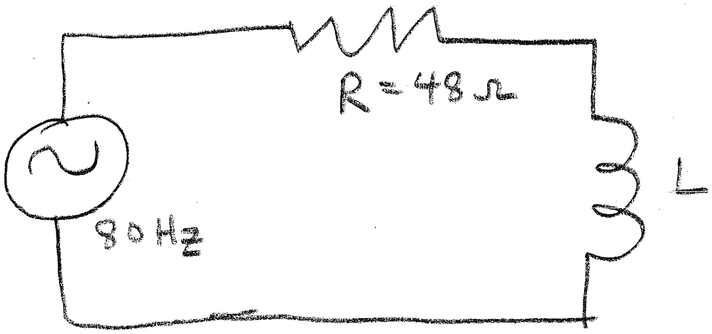
$v = v_R + v_L + v_C$

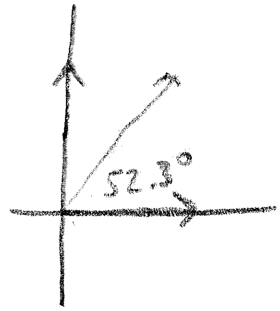
c) $t = 40\,ms = 0.04\,s$

$v = -23.76\,V$ $v_R = -8.37\,V$ $v_L = 2.72\,V$, $v_C = -18.1$

$v = v_R + v_L + v_C$

31.37 A coil has a resistance of 48.0 Ω. At a frequency of 80.0 Hz the voltage across the coil leads the current in it by 52.3°. Determine the inductance of the coil.

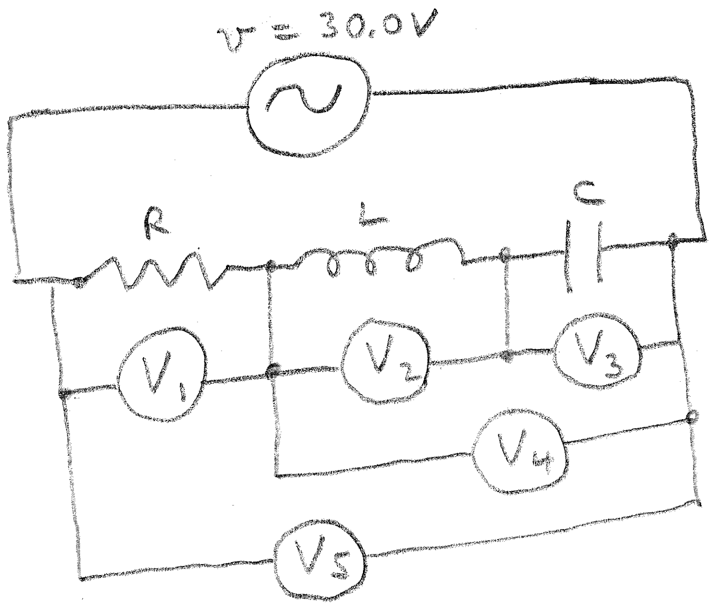




$$\tan\theta = \frac{X_L}{R} = \frac{L\omega}{R} \Rightarrow L = \frac{R}{\omega}\tan\theta$$

$$= \frac{48\,\Omega}{80\,\frac{1}{s}\left(\frac{2\pi\,\text{rad}}{\text{cyc}}\right)}\tan 52.3^\circ = \boxed{0.1236\text{ H}}$$

31.38 Five infinite-impedance voltmeters, calibrated to read rms values, are connected as shown in Fig 31.21. Let $R = 200\ \Omega$, $L = 0.400$ H, $C = 6.00\ \mu F$, and $V = 30.0$ V. What is the reading of each voltmeter if a) $\omega = 200$ rad/s; b) $\omega = 1000$ rad/s?



$X_L = \omega L = 80\Omega$

$X_C = \frac{1}{\omega C} = 833.\overline{3}\ \Omega$

$R = 200\Omega$

a) $I = \frac{V}{Z}$ $\quad Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$$Z = \sqrt{(200\Omega)^2 + \left(200\,\tfrac{rad}{s}(.4H) - \frac{1}{(200\,\frac{rad}{s})(6\times10^{-6}F)}\right)^2}$$

$= 779.43\ \Omega$

$I = \frac{V}{Z} = 0.0384897\ A$

rms $\quad V_1 = \frac{IR}{\sqrt{2}} = 5.44\ V \qquad V_2 = \frac{I X_L}{\sqrt{2}} = 2.18\ V$

$V_3 = \frac{I X_C}{\sqrt{2}} = 22.68\ V \qquad V_4 = \frac{I(X_C - X_L)}{\sqrt{2}} = 20.5\ V$

$V_5 = \frac{30}{\sqrt{2}} = 21.2\ V$

31.38 b) $\omega = 1000 \text{ rad/s}$

$X_L = \omega L = 400\,\Omega$ $X_c = 166.\bar{6}\,\Omega$

$Z = \sqrt{R^2 + (X_L - X_c)^2} = 307.3\,\Omega$

$I = \frac{V}{Z} = 0.0976187 \text{ A}$

$V_1 = \frac{IR}{\sqrt{2}} = 13.8 \text{ V}$ $V_2 = \frac{I X_L}{\sqrt{2}} = 27.6 \text{ V}$

$V_3 = \frac{I X_c}{\sqrt{2}} = 11.5 \text{ V}$ $V_4 = \frac{|I X_c - I X_L|}{\sqrt{2}} = 16.1 \text{ V}$

$V_5 = \frac{30 \text{ V}}{\sqrt{2}} = 21.2 \text{ V}$

31.46 A Low-Pass Filter. Figure 31.23 shows a low-pass filter (see Problem 31.45); the output voltage is taken across the capacitor in an *L-R-C* series circuit. Derive an expression for V_{out}/V_s, the ratio of the output and source voltage amplitudes, as a function of the angular frequency ω of the source. Show that when ω is large this ratio is proportional to ω^{-2} and thus is very small, and show that the ratio approaches unity in the limit of small frequency.

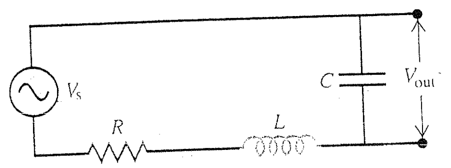


Figure 31.23 Problem 31.46.

a) $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_s}{Z} \qquad V_{out} = I X_C = \frac{V_s}{Z} X_C = \frac{V_s \frac{1}{\omega C}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\Rightarrow \frac{V_{out}}{V_s} = \boxed{\frac{1}{\omega C}\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2}}$$

for ω large $$\frac{V_{out}}{V_s} = \frac{1}{\omega C}\,\frac{1}{\omega L}\,\frac{1}{\sqrt{\left(\frac{R}{\omega L}\right)^2 + \left(1 - \frac{1}{\omega^2 LC}\right)^2}}$$

$$\Rightarrow \frac{V_{out}}{V_s} = \frac{1}{\omega^2 LC}\,\frac{1}{\sqrt{\underbrace{\left(\frac{R}{\omega L}\right)^2}_{\text{small}} + \left[1 - \underbrace{\left(\frac{1}{\omega^2 LC}\right)}_{\text{small}}\right]^2}} \cong \frac{1}{\omega^2}\,\frac{1}{LC}\,\frac{1}{\sqrt{1}}$$

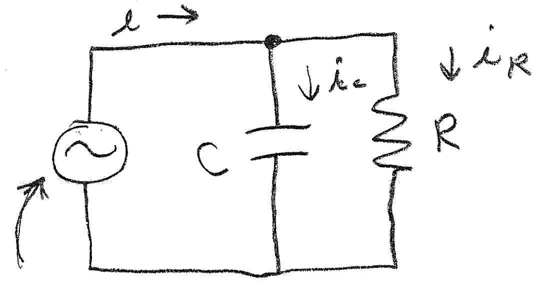
$\omega L >> R$

$\omega^2 >> \frac{1}{LC}$ $$\Rightarrow \frac{V_{out}}{V_s} \propto \frac{1}{\omega^2}$$

for small ω

$$\frac{V_{out}}{V_s} = \frac{1}{\omega C}\,\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{\underbrace{R^2\omega^2 C^2}_{\text{small}} + (\underbrace{\omega^2 LC}_{\text{small}} - 1)^2}} \sim 1$$

31.52 A 400-Ω resistor and a 6.00-μF capacitor are connected in parallel to an ac generator that supplies an rms voltage of 220 V at an angular frequency of 360 rad/s. Use the results of Problem 31.50. Note that since there is no inductor in the circuit, the $1/\omega L$ term is not present in the expression for Z. Find a) the current amplitude in the resistor; b) the current amplitude in the capacitor; c) the phase angle of the source current with respect to the source voltage; d) the amplitude of the current through the generator. e) Does the source current lag or lead the source voltage?

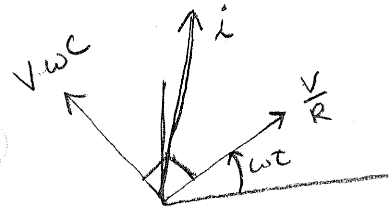


$v = V\cos\omega t$

$$\text{KCR} \Rightarrow \quad i = i_c + i_R = C\frac{dv}{dt} + \frac{v}{R}$$

$$\Rightarrow \quad i = -C\omega V\sin\omega t + \frac{V\cos\omega t}{R}$$

$$\Rightarrow i \equiv I\cos(\omega t + \phi) = -\omega C V\sin\omega t + \frac{V}{R}\cos\omega t$$
$$= \omega C V\cos(\omega t + 90^\circ) + \frac{V}{R}\cos\omega t$$



$$i = \underbrace{\sqrt{\underbrace{(\omega C V)^2}_{I_c} + \underbrace{\left(\frac{V}{R}\right)^2}_{I_R}}}_{I}\ \cos\left[\omega t + \tan^{-1}\left(\frac{\omega C}{\frac{1}{R}}\right)\right]$$

$$\tan\phi = \frac{\omega C V}{V/R} = \frac{I_c}{I_R}$$

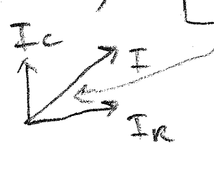
a)

$$\Rightarrow I_R = \frac{V}{R} = \frac{\sqrt{2}\,(220\,V)}{400\,\Omega} = \boxed{0.778\ A}$$

b) $i_c = \omega C V\sin\omega t \Rightarrow I_c = \omega C V = \left(360\ \frac{rad}{s}\right)(6\times10^{-6}\,F)(\sqrt{2})(220\,V)$

$I_c = \boxed{0.672\ A}$

c) $$\phi = \tan^{-1}\left(\frac{\omega C}{1/R}\right) = \tan^{-1}\left((360)(6\times10^{-6})(400)\right) = \boxed{40.8^\circ}$$



d) $I = \sqrt{I_c^2 + I_R^2} = \boxed{1.03\,A}$

e) i leads v