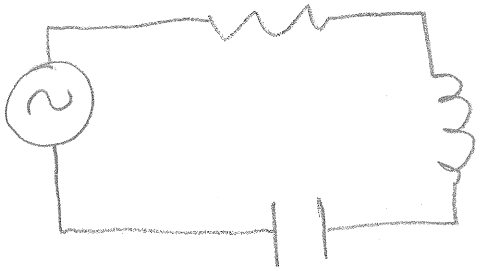
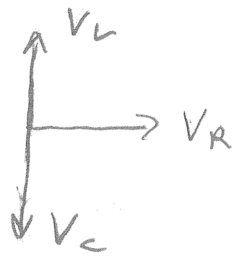
HW 23

From Text book

31.21 In an *L*-*R*-*C* series circuit, the rms voltage across the resistor is 30.0 V, across the capacitor it is 90.0 V and across the inductor it is 50.0 V. What is the rms voltage of the source?





Phase relation

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{30^2 + (90-50)^2}\ V$$

$$= \sqrt{50^2}\ V = \boxed{50\ V}$$

31.31 Consider an L-R-C series circuit composed of the inductor, resistor, and capacitor described in Exercise 31.12. The circuit is connected to an ac source with voltage amplitude 30.0 V. a) At what frequency (in Hz) is the circuit in resonance? b) Sketch the phasor diagram at the resonance frequency. c) What is the reading of each voltmeter shown in Fig. 31.21 when the source frequency equals the resonance frequency? The voltmeters are calibrated to read rms voltages. d) What is the resonance frequency if the resistance is reduced to 100 Ω? e) What is then the rms current at resonance?

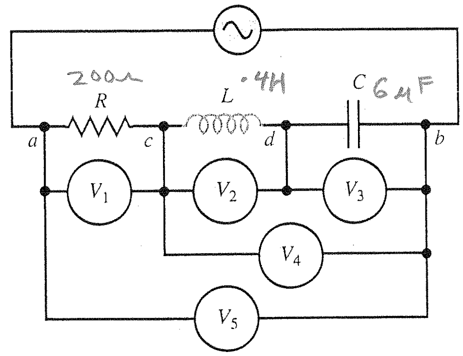


Figure 31.21 Exercise 31.31 and Problem 31.38.

a) at resonance $\omega \to \omega_0$

$X_L = X_C$

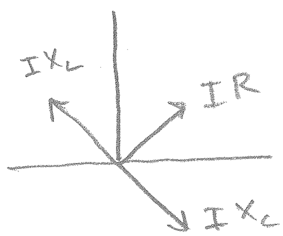
$\Rightarrow \omega_0 L = \dfrac{1}{\omega_0 C}$

$\Rightarrow \omega_0^2 = \dfrac{1}{LC}$

$\omega_0^2 = \dfrac{1}{(.4H)(6\times10^{-6}F)}$

$\Rightarrow \boxed{\omega_0 = 645.5\ Hz}$

b)



c) At resonance $Z = \sqrt{R^2 + \underbrace{(X_L - X_C)^2}_{0}} = R$

$\Rightarrow I_{rms} = \dfrac{V}{Z} = \dfrac{V}{R} = \dfrac{30\,V}{\sqrt{2}}\ \dfrac{1}{200\,\Omega} = \boxed{0.106\,A}$ 0.106066 A

$V_1 = I_{rms} R = \boxed{21.21\ V}$

$V_2 = I_{rms} X_L = I_{rms}\,\omega L = \boxed{27.4\,V}$

$V_3 = V_2 = \boxed{27.4\ V}$

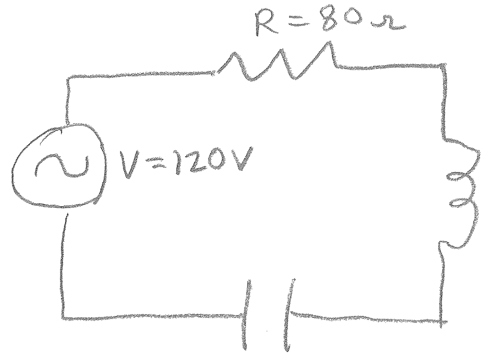
$V_4 = \boxed{0}$ at resonance

$V_5 = \boxed{21.21\ V}$

d) same $\omega_0 = \boxed{645.5\,Hz}$

e) $I_{rms} = \dfrac{V}{Z} = \dfrac{V}{R} = \dfrac{30V}{\sqrt{2}}\ \dfrac{1}{100\,\Omega} = \boxed{0.212\ A}$

31.59 In an *L-R-C* series circuit, the source has a voltage amplitude of 120 V, $R = 80.0\ \Omega$, and the reactance of the capacitor is 480 Ω. The voltage amplitude across the capacitor is 360 V. a) What is the current amplitude in the circuit? b) What is the impedance? c) What two values can the reactance of the inductor have? d) For which of the two values found in part (c) is the angular frequency less than the resonance angular frequency? Explain.



$X_c = 480\,\Omega$

$V_c = 360\,V$

a) $I_c = \dfrac{V_c}{X_c} = \dfrac{360\,V}{480\,\Omega} = \boxed{0.75\,A}$

b) $Z_T = \dfrac{V}{I} = \dfrac{V}{I_c} = \dfrac{120\,V}{(0.75\,A)} = \boxed{160\,\Omega}$

c) $Z_T = \sqrt{R^2 + (X_c - X_L)^2} \Rightarrow (X_c - X_L)^2 = Z_T^2 - R^2$

$\Rightarrow X_c - X_L = \pm\sqrt{Z_T^2 - R^2} \Rightarrow X_L = X_c \pm \sqrt{Z_T^2 - R^2}$

$= 480\,\Omega \pm \sqrt{(160)^2 - (80)^2}\,\Omega = (480 \pm 138.56406)\,\Omega$

$= \boxed{341\,\Omega\ ,\ 618.6\,\Omega}$

$\omega < \omega_0 \qquad \omega > \omega_0$

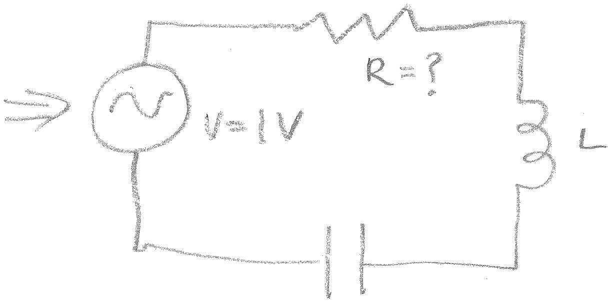
d) $\boxed{341\,\Omega}$ $\qquad X_L = \omega L$

31.60 Designing an FM Radio Receiver. You enjoy listening to KONG-FM, which broadcasts at 94.1 MHz. You detest listening to KRUD-FM, which broadcasts at 94.0 MHz. You live the same distance from both stations and both transmitters are equally powerful, so both radio signals produce the same 1.0-V source voltage as measured at your house. Your goal is to design an *L-R-C* radio circuit with the following properties: i) It gives the maximum power response to the signal from KONG-FM; ii) the average power delivered to the resistor in response to KRUD-FM is 1.00% of the average power in response to KONG-FM. This limits the power received from the unwanted station, making it inaudible. You are required to use an inductor with $L = 1.00\ \mu H$. Find the capacitance C and resistance R that satisfy the design requirements.

$$P_{ave} = \frac{1}{2} V I \cos\phi$$

$$= \frac{1}{2} V \frac{V}{Z}\left(\frac{R}{Z}\right)$$

$$= \frac{1}{2} V^2 \frac{R}{Z^2} = \frac{1}{2} V^2 \frac{R}{R^2+X^2}$$



P_{av} is max if $X = 0$

$$\Rightarrow X = X_L - X_C = 0$$

$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}$$

resonance gives max P_{av}

$\omega_0 = 94.1\ MHz$ $C = ?$

$\omega_1 = 94.0\ MHz$

① $P_{ave_{max}}(\omega_0) = 100\ P_{ave}(\omega_1)$ & ② $\omega_0^2 = \frac{1}{LC}$

$\Rightarrow$ 2 Equation, 2 unknowns

② $\Rightarrow C = \frac{1}{L\omega_0^2} = \frac{1}{(1\times10^{-6}\ H)(94.1\times10^6\ 1/s)^2\left(\frac{2\pi\ rad}{rev}\right)^2} = \boxed{2.86\times10^{-12}\ F}$

① $\Rightarrow \frac{P_{ave_{max}}(\omega_0)}{P_{ave}(\omega_1)} = \frac{R^2 + X^2(\omega_1)}{R^2 + \cancel{X^2(\omega_0)}} \Rightarrow R^2\left(\frac{P_{ave}(\omega_0)}{P_{ave}(\omega_1)} - 1\right) = X^2$

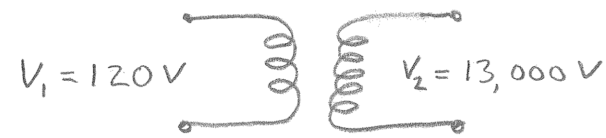
$$\Rightarrow R^2\ 99 = X^2 \Rightarrow R = \frac{1}{\sqrt{99}} X(\omega_1) = \frac{1}{99}\left[\frac{1}{\omega_1 C} - \omega L\right]$$

$$= \frac{1}{\sqrt{99}}\left[\frac{1}{(94.0\times10^6\ \frac{1}{s})(2\pi)(2.86\times10^{-12}\ F)} - 94.0\times10^6\ \frac{1}{s}(2\pi)\ 1\times10^{-6}\ H\right]$$

$$= \boxed{0.126\ \Omega} \qquad \Rightarrow I = \frac{V}{R} = 8\ A$$

not good

31.34 A Step-Up Transformer. A transformer connected to a 120-V (rms) ac line is to supply 13,000 V (rms) for a neon sign. To reduce shock hazard, a fuse is to be inserted in the primary circuit; the fuse is to blow when the rms current in the secondary circuit exceeds 8.50 mA. a) What is the ratio of secondary to primary turns of the transformer? b) What power must be supplied to the transformer when the rms secondary current is 8.50 mA? c) What current rating should the fuse in the primary circuit have?

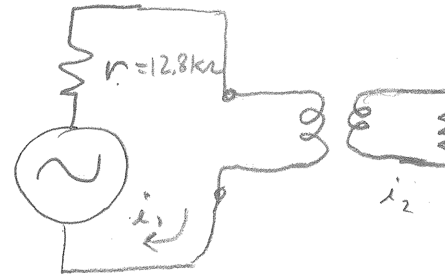


a) $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1} = \dfrac{13{,}000\,V}{120\,V} = \boxed{108.\overline{3}}$

b) $P_1 = P_2 = V_2 I_2 = (13{,}000\,V)(8.5\times10^{-3}\,A) = \boxed{110.5\,W}$

c) $I_1 = \dfrac{P_1}{V_1} = \dfrac{P_2}{V_1} = \dfrac{110.5\,W}{120\,V} = \boxed{0.921\,A}$

31.35 The internal resistance of an ac source is 12.8 kΩ. a) What should the ratio of primary to secondary turns of a transformer be to match the source to a load with a resistance of 8.00 Ω? ("Matching" means that the effective load resistance equals the internal resistance of the source. See Exercise 31.33.) b) If the voltage amplitude of the source is 60.0 V, what is the voltage amplitude in the secondary circuit under open circuit conditions?

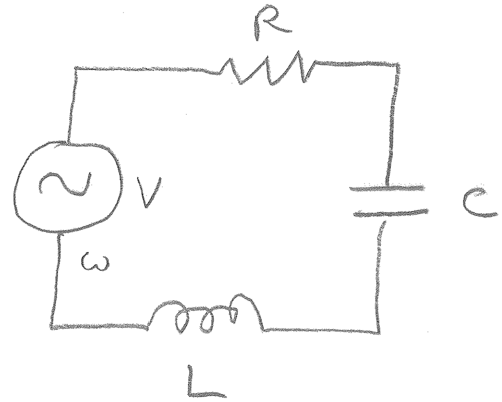


a) $$r = \frac{V_1}{I_1} = \frac{\left(\frac{N_1}{N_2} V_2\right)}{\left(\frac{N_2}{N_1} I_2\right)} = \left(\frac{N_1}{N_2}\right)^2 \frac{V_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 R$$

$$\Rightarrow \frac{N_1}{N_2} = \sqrt{\frac{r}{R}} = \sqrt{\frac{12.8 \times 10^3\,\Omega}{8\,\Omega}} = \boxed{40}$$

b) $$V_2 = \frac{N_2}{N_1} V_1 = \frac{60\,V}{40} = \boxed{1.5\,V}$$

31.64 A resistance R, capacitance C, and inductance L are connected in series to a voltage source with amplitude V and variable angular frequency ω. If $\omega = \omega_0$, the resonance angular frequency, find a) the maximum current in the resistor; b) the maximum voltage across the capacitor; c) the maximum voltage across the inductor; d) the maximum energy stored in the capacitor; and e) the maximum energy stored in the inductor. Give your answers in terms of R, C, L, and V.



a) $\omega = \omega_0 \Rightarrow X_C = X_L, \quad Z = R$

$$I = \frac{V}{Z} = \boxed{\frac{V}{R}} \qquad \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \sqrt{\frac{1}{LC}}$$

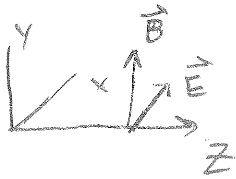
b) $$V_C = I X_C = I \frac{1}{\omega C} = I \frac{\sqrt{LC}}{C} = \boxed{\frac{V}{R}\sqrt{\frac{L}{C}}}$$

c) $$V_L = I X_L = I X_C = \boxed{\frac{V}{R}\sqrt{\frac{L}{C}}}$$

d) $$U_{C_{max}} = \frac{1}{2} C V_{C_{max}}^2 = \frac{1}{2} C \left(\frac{V}{R}\right)^2 \left(\frac{L}{C}\right) = \boxed{\frac{1}{2} L \frac{V^2}{R^2}}$$

e) $$U_{L_{max}} = \frac{1}{2} L I^2 = \boxed{\frac{1}{2} L \frac{V^2}{R^2}}$$

32.3 A sinusoidal electromagnetic wave of frequency 6.10×10^{14} Hz travels in vacuum in the $+z$-direction. The $\vec{B}$ field is parallel to the y-axis and has amplitude 5.80×10^{-4} T. Write the vector equations for $\vec{E}(z, t)$ and $\vec{B}(z, t)$.



$\vec{E} \times \vec{B}$ = direction

a) $\vec{E} = c B_{max} \cos(kz - \omega t)\,\hat{i} = c\,B_{max} \cos\left(\frac{2\pi f}{c} z - 2\pi f t\right)$

$= 3\times10^{8}\,\text{m/s}\,(5.8\times10^{-4}\,\text{T}) \cos\left(2\pi \frac{6.10\times10^{14}\,\frac{1}{s}}{3\times10^{8}\,\text{m/s}} z - 2\pi\, 6.1\times10^{14}/\text{s}\ t\right)$

$$\boxed{\vec{E} = 1.74\times10^{5}\,\frac{V}{m} \cos\left(1.278\times10^{7}\left(\frac{rad}{m}\right) z - 3.833\times10^{15}\left(\frac{rad}{s}\right) t\right)\hat{i}}$$

$$\boxed{\vec{B} = 5.8\times10^{-4}\,T \cos\left(1.278\times10^{7}\,\frac{rad}{m}\, z - 3.833\times10^{15}\,\frac{rad}{s}\, t\right)\hat{j}}$$

32.10 An electromagnetic wave with frequency 65.0 Hz travels in an insulating magnetic material that has dielectric constant 3.64 and relative permeability 5.18 at this frequency. The electric field has amplitude 7.20×10^{-3} V/m. a) What is the speed of propagation of the wave? b) What is the wavelength of the wave? c) What is the amplitude of the magnetic field? d) What is the intensity of the wave?

$K_D = 3.64$ $\quad E_{max} = 7.20 \times 10^{-3} \frac{V}{m}$

$K_m = 5.18$

a) $$v = \frac{1}{\sqrt{\epsilon \mu}} = \frac{1}{\sqrt{K_D \epsilon_0 K_m \mu_0}} = \frac{c}{\sqrt{K_D K_m}}$$

$$= \frac{3 \times 10^8 \, m/s}{\sqrt{(3.64)(5.18)}} = \boxed{6.91 \times 10^7 \, m/s}$$

b) $$\lambda = \frac{v}{f} = \frac{6.91 \times 10^7 \, m/s}{65 \, Hz} = \boxed{1.06 \times 10^6 \, m}$$

c) $$B = \frac{E}{v} = \frac{7.2 \times 10^{-3} \frac{V}{m}}{6.91 \times 10^7 \, m/s} = \boxed{1.04 \times 10^{-10} \, T}$$

d) $$I = \frac{E_{max} B_{max}}{2 \mu} = \frac{E_{max}^2}{v} \frac{1}{2 K_D \mu_0} = \frac{(7.2 \times 10^{-3} \frac{V}{m})^2}{6.91 \times 10^7 \, m/s} \frac{1}{2} \frac{1}{5.18} \frac{1}{4\pi \times 10^{-7} \, Tm/A}$$

$$= \boxed{5.76 \times 10^{-8} \, W/m^2}$$

32.14 A sinusoidal electromagnetic wave from a radio station passes perpendicularly through an open window that has area 0.500 m^2. At the window, the electric field of the wave has rms value 0.0200 V/m. How much energy does this wave carry through the window during a 30.0-s commercial?

$$I = \frac{1}{2} \epsilon_0 c E_{max}^2 \qquad U = I A (\Delta t)$$

$$\Rightarrow \quad U = \left(\frac{1}{2} \epsilon_0 c E_{max}^2\right) A (\Delta t)$$

$$= \frac{1}{2}\left(8.85\times10^{-12} \frac{\cancel{C^2}}{Nm^2}\right)\left(3\times10^8 \, m/s\right)\left(\sqrt{2}\right)^2\left(.02 \, Ks\left(\frac{m}{s}\right)^2 \frac{1}{\cancel{C}} m\right)^2\left(.5 \, m^2\right)\left(30 \, s\right)$$

$$= \boxed{1.593\times10^{-5} \text{ Jouls}}$$

32.22 In the 25-ft Space Simulator facility at NASA's Jet Propulsion Laboratory, a bank of overhead arc lamps can produce light of intensity 2500 W/m^2 at the floor of the facility. (This simulates the intensity of sunlight near the planet Venus.) Find the average radiation pressure (in pascals and in atmospheres) on a) a totally absorbing section of the floor; b) a totally reflecting section of the floor. c) Find the average momentum density (momentum per unit volume) in the light at the floor.

d) $$P_{rad} = \frac{I}{c} = \frac{2500\ W/m^2}{3 \times 10^8\ m/s} = \boxed{8.\bar{3} \times 10^{-6}\ Pa}$$

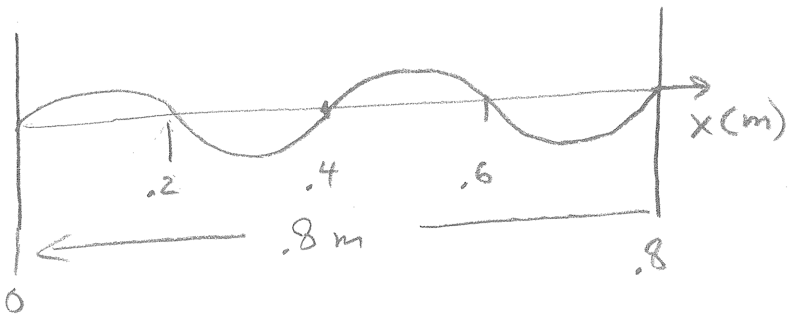
$$= 8.\bar{3} \times 10^{-6}\ Pa \ \frac{1\ atm}{1.013 \times 10^5\ Pa} \cong \boxed{8.23 \times 10^{-11}\ atm}$$

b) $$P_{rad\,ref} = 2\frac{I}{c} = \boxed{1.66 \times 10^{-5}\ Pa \cong 1.65 \times 10^{-10}\ atm}$$

c) $$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} \Rightarrow \frac{dp}{dV} = \frac{1}{A\,c}\frac{dp}{dt} = \frac{S}{c^2}$$

$$\left.\frac{dp}{dV}\right|_{ave} = \frac{S_{ave}}{c^2} = \frac{2500\ W/m^2}{(3 \times 10^8\ m/s)^2} = \boxed{2.778 \times 10^{-14}\ \frac{kg}{m^2 s}}$$

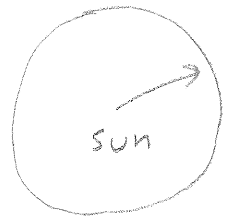
32.28 An electromagnetic standing wave in air of frequency 750 MHz is set up between two conducting planes 80.0 cm apart. At which positions between the planes could a point charge be placed at rest so that it would *remain* at rest? Explain.



$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \,m/s}{750 \times 10^6 \,1/s} = \underline{0.4\,m}$$

At positions .2m, .4m, .6m from the ends

32.41 The sun emits energy in the form of electromagnetic waves at a rate of 3.9×10^{26} W. This energy is produced by nuclear reactions deep in the sun's interior. a) Find the intensity of electromagnetic radiation and the radiation pressure on an absorbing object at the surface of the sun (radius $r = R = 6.96 \times 10^5$ km) and at $r = R/2$, in the sun's interior. Ignore any scattering of the waves as they move radially outward from the center of the sun. Compare to the values given in Section 32.4 for sunlight just before it enters the earth's atmosphere. b) The gas pressure at the sun's surface is about 1.0×10^4 Pa; at $r = R/2$, the gas pressure is calculated from solar models to be about 4.7×10^{13} Pa. Comparing with your results in part (a), would you expect that radiation pressure is an important factor in determining the structure of the sun? Why or why not?



a) $$I = \frac{\text{Power}}{\text{Area}} = \frac{3.9 \times 10^{26}\ \text{W}}{4\pi (6.96 \times 10^5 \times 10^3\ \text{m})^2} = \boxed{6.407 \times 10^7\ \frac{\text{W}}{\text{m}^2}}$$

$$P_{rad} = \frac{S}{c} = \frac{I}{c} = \frac{6.407 \times 10^7\ \frac{\text{W}}{\text{m}^2}}{(3 \times 10^8\ \text{m/s})} = \boxed{2.13 \times 10^{-1}\ \text{Pa}}$$

$$I \propto \frac{1}{r^2} \Rightarrow I' = I\left(\frac{r}{r'}\right)^2 = I\left(\frac{2}{1}\right)^2 = 4I$$

$$P'_{rad} = 4P_{rad} = \boxed{8.54 \times 10^{-1}\ \text{Pa}}$$ half way to center

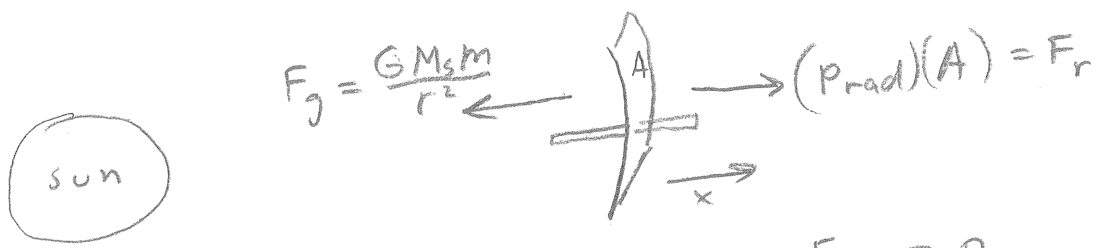
Section 32.4 $P_{rad} = 4.7 \times 10^{-6}$ Pa

c) No. surface pressure from radiation is much small in Both cases.

32.52 NASA is giving serious consideration to the concept of *solar sailing*. A solar sailcraft uses a large, low-mass sail and the energy and momentum of sunlight for propulsion. a) Should the sail be absorbing or reflective? Why? b) The total power output of the sun is 3.9×10^{26} W. How large a sail is necessary to propel a 10,000-kg spacecraft against the gravitational force of the sun? Express your result in square kilometers. c) Explain why your answer to part (b) is independent of the distance from the sun.

a) reflective so there is more momentum transfered.

b)



$$\sum F_x = F_g - F_r = 0$$

$$\Rightarrow \frac{G M_s m}{r^2} = (P_{rad}) A = \frac{\frac{2\,\text{Power}}{4\pi r^2}}{c} A$$

$$\Rightarrow G M_s m = \frac{2(\text{Power})}{4\pi c} A \Rightarrow A = \frac{G M_s m \, 2\pi \, c}{\text{Power}}$$

$$A = \frac{(6.7\times 10^{-11} \frac{\cancel{N} m^2}{\cancel{kg^2}})(2\times 10^{30} \cancel{kg})(10^4 \cancel{kg})(3\times 10^8 \cancel{m/s})\, 2\pi}{3.9\times 10^{26} \frac{\cancel{Nm}}{\cancel{s}}}$$

$$= 6.48 \times 10^6 \, m^2 \left(\frac{1\text{ km}}{1000\text{ m}}\right)^2 = 6.48\,(\text{km})^2$$

c) $F_g \propto \frac{1}{r^2}$ $\quad F_r \propto \frac{1}{r^2}$ $\quad \frac{1}{r^2}$ factor cancels