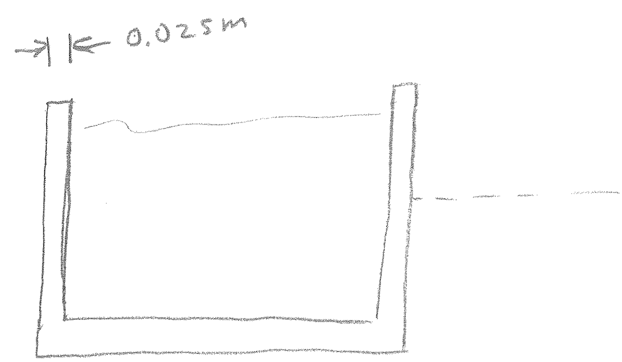
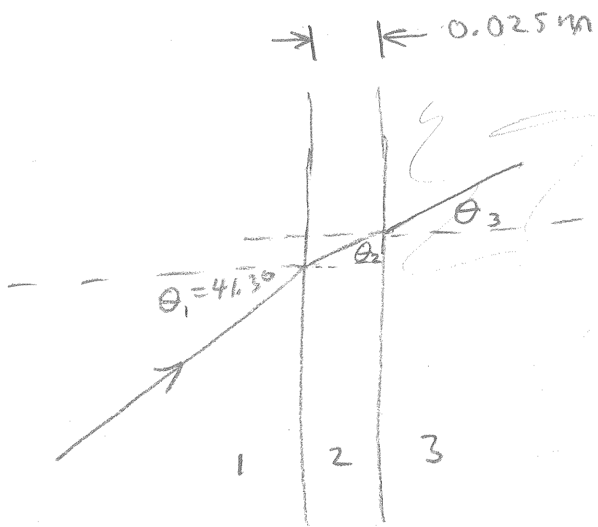
From Text

HW 24

33.8 a) A tank containing methanol has walls 2.50 cm thick made of glass of refractive index 1.550. Light from the outside air strikes the glass at a 41.3° angle with the normal to the glass. Find the angle the light makes with the normal in the methanol. b) The tank is emptied and refilled with an unknown liquid. If light incident at the same angle as in part (a) enters the liquid in the tank at an angle of 20.2° from the normal, what is the refractive index of the unknown liquid?





a) $\overset{1 \text{ air}}{\cancel{n_1}} \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$

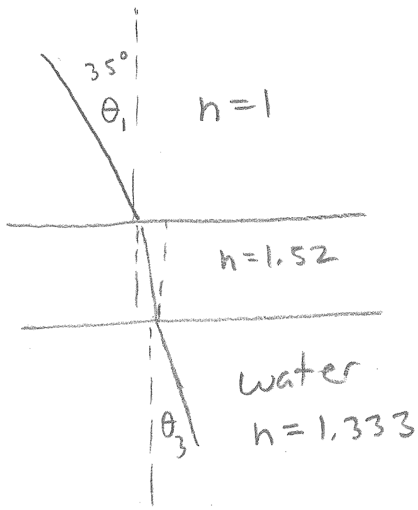
$\Rightarrow \sin\theta_1 = n_{meth} \sin\theta_3 \Rightarrow \theta_3 = \sin^{-1}\left(\frac{\sin\theta_1}{n_{meth}}\right)$

$\Rightarrow \theta_{meth} = \sin^{-1}\left(\frac{\sin 41.3°}{1.329}\right) = \boxed{29.8°}$

b) $n_3 = \frac{\sin\theta_1}{\sin\theta_3} = \frac{\sin 41.3°}{\sin 20.2°} = \boxed{1.91}$

33.10 A horizontal, parallel-sided plate of glass having a refractive index of 1.52 is in contact with the surface of water in a tank. A ray coming from above in air makes an angle of incidence of 35.0° with the normal to the top surface of the glass. a) What angle does the ray refracted into the water make with the normal to the surface? b) What is the dependence of this angle on the refractive index of the glass?

a)



$$\sin\theta_1 = n_3 \sin\theta_3$$

$$\Rightarrow \theta_3 = \sin^{-1}\left(\frac{\sin\theta_1}{n_3}\right) = \sin^{-1}\left(\frac{\sin 35^\circ}{1.333}\right)$$

$$\Rightarrow \theta_3 = \boxed{25.5^\circ}$$

b) No dependence,

33.18 Light is incident along the normal on face AB of a glass prism of refractive index 1.52, as shown in Fig. 33.36. Find the largest value the angle α can have without any light refracted out of the prism at face AC if: a) the prism is immersed in air; b) the prism is immersed in water.

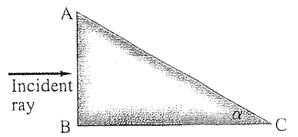
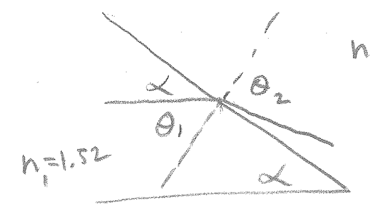


Figure 33.36 Exercise 33.18.

a)



$n_2 = 1$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_1 = \theta_c \quad \theta_2 = 90^\circ$

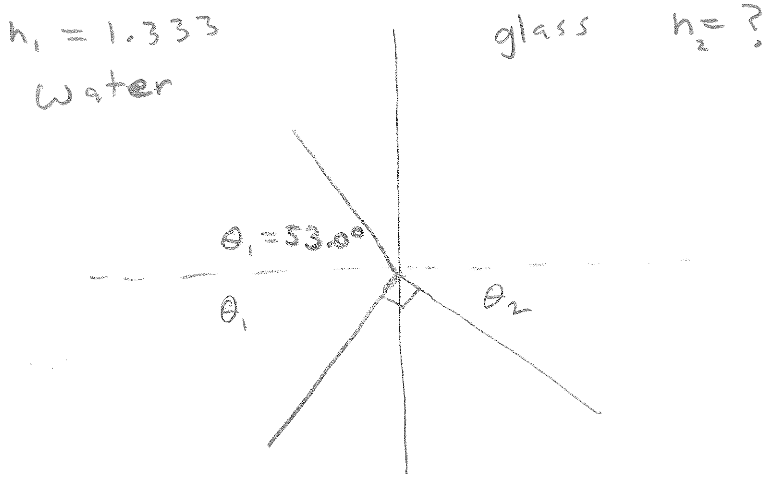
$\sin\theta_c = \frac{1}{n_1} \Rightarrow \cos\alpha = \frac{1}{n_1} \Rightarrow \alpha = \cos^{-1}\left(\frac{1}{1.52}\right)$

$\Rightarrow \alpha = \boxed{48.9^\circ}$

b) $n_2 = 1.333$ water

$\cos\alpha = \frac{n_2}{n_1} \Rightarrow \alpha = \cos^{-1}\frac{1.333}{1.52} = \boxed{28.7^\circ}$

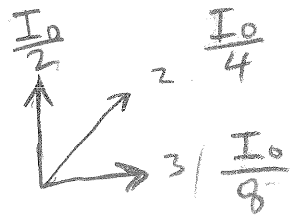
33.22 Light traveling in water strikes a glass plate at an angle of incidence of 53.0°; part of the beam is reflected and part is refracted. If the reflected and refracted portions make an angle of 90.0° with each other, what is the index of refraction of the glass?

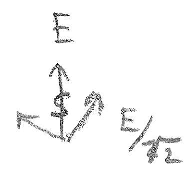


$$\theta_2 + 90° + \theta_1 = 180° \Rightarrow \theta_2 = 90° - \theta_1 = 37.0°$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2} = 1.333 \frac{\sin 53°}{\sin 37°} = \boxed{1.77}$$

33.29 Three Polarizing Filters. Three polarizing filters are stacked with the polarizing axes of the second and third at 45.0° and 90.0°, respectively, with that of the first. a) If unpolarized light of intensity I_0 is incident on the stack, find the intensity and state of polarization of light emerging from each filter. b) If the second filter is removed, what is the intensity of the light emerging from each remaining filter?





$E \rightarrow \frac{E}{\sqrt{2}}$

$I \rightarrow \frac{E^2}{2} = \frac{I}{2}$

a)

$$I = \boxed{\frac{I_0}{8} \text{ @ } 90^\circ}$$

b) $I_1 = \frac{I_0}{2}$ $\quad I_3 = 0$

33.40 After a long day of driving you take a late-night swim in a motel swimming pool. When you go to your room, you realize that you have lost your room key in the pool. You borrow a powerful flashlight and walk around the pool, shining the light into it. The light shines on the key, which is lying on the bottom of the pool, when the flashlight is held 1.2 m above the water surface and is directed at the surface a horizontal distance of 1.5 m from the edge (see Fig. 33.39). If the water here is 4.0 m deep, how far is the key from the edge of the pool?

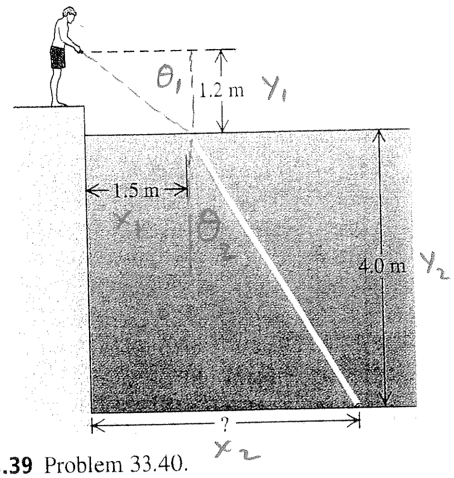


Figure 33.39 Problem 33.40.

$$\sin\theta_1 = n_w \sin\theta_2$$

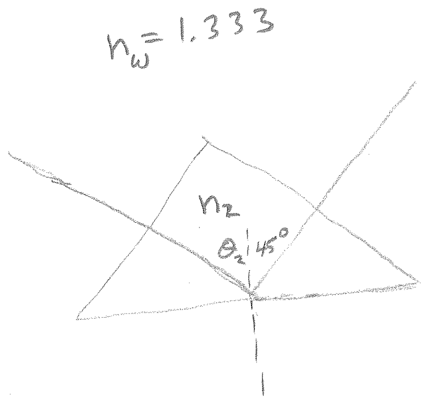
$$\Rightarrow \frac{X_1}{\sqrt{X_1^2+Y_1^2}} = n_w \frac{X_2}{\sqrt{X_2^2+Y_2^2}}$$

$$\Rightarrow \sqrt{X_2^2+Y_2^2}\,\sin\theta_1 = n_w X_2 \Rightarrow X_2^2 + Y_2^2 = \left(\frac{n_w X_2}{\sin\theta_1}\right)^2$$

$$\Rightarrow X_2^2\left(\frac{n_w}{\sin\theta_1} - 1\right) = Y_2^2 \Rightarrow X_2^2 = \frac{Y_2^2}{\left(\frac{n_w}{\sin\theta_1} - 1\right)}$$

$$\Rightarrow X_2 = \frac{Y_2}{\sqrt{\frac{n_w}{\sin\theta_1} - 1}} = \frac{4\text{ m}}{\sqrt{\frac{1.333}{\left(\frac{1.5}{\sqrt{(1.2)^2+(1.5)^2}}\right)} - 1}} \simeq \boxed{4.49\text{ m}}$$

33.44 A 45°–45°–90° prism is immersed in water. A ray of light is incident normally on one of its shorter faces. What is the minimum index of refraction that the prism must have if this ray is to be totally reflected within the glass at the long face of the prism?



$$n_w \sin 90° = n_2 \sin\theta_2 \Rightarrow n_2 = \frac{n_w}{\sin\theta_2} = \frac{1.333}{1/\sqrt{2}} = \sqrt{2}(1.333)$$

$$= \boxed{1.88}$$

33.46 Light is incident normally on the short face of a 30°–60°–90° prism (see Fig. 33.42). A drop of liquid is placed on the hypotenuse of the prism. If the index of the prism is 1.62, find the maximum index that the liquid may have if the light is to be totally reflected.

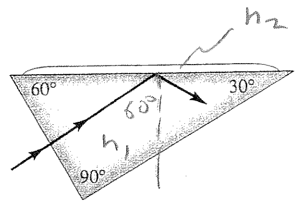


Figure 33.42 Problem 33.46.

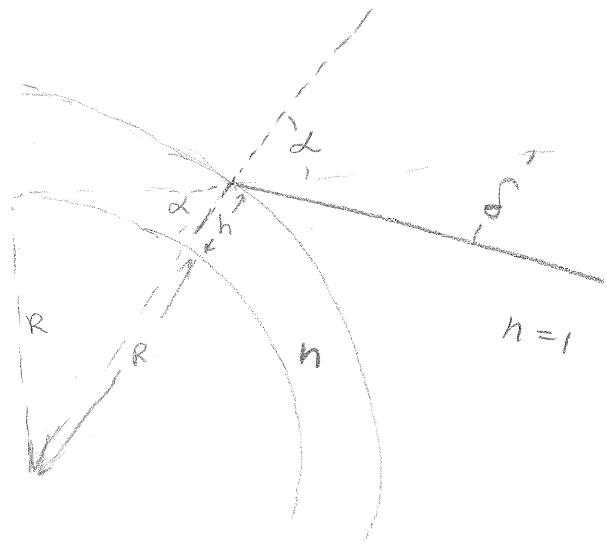
$$n_1 \sin 60° = n_2 \sin 90° \Rightarrow n_2 = n_1 \sin 60°$$

$$= 1.62\left(\frac{\sqrt{3}}{2}\right) = \boxed{1.40}$$

33.49 When the sun is either rising or setting and appears to be just on the horizon, it is in fact *below* the horizon. The explanation for this seeming paradox is that light from the sun bends slightly when entering the earth's atmosphere, as shown in Fig. 33.44. Since our perception is based on the idea that light travels in straight lines, we perceive the light to be coming from an apparent position that is an angle δ above the sun's true position. a) Make the simplifying assumptions that the atmosphere has uniform density, and hence uniform index of refraction n, and extends to a height h above the earth's surface, at which point it abruptly stops. Show that the angle δ is given by

$$\delta = \arcsin\left(\frac{nR}{R+h}\right) - \arcsin\left(\frac{R}{R+h}\right)$$

where $R = 6378$ km is the radius of the earth. b) Calculate δ using $n = 1.0003$ and $h = 20$ km. How does this compare to the angular radius of the sun, which is about one quarter of a degree? (In actuality a light ray from the sun bends gradually, not abruptly, since the density and refractive index of the atmosphere change gradually with altitude.)



a)

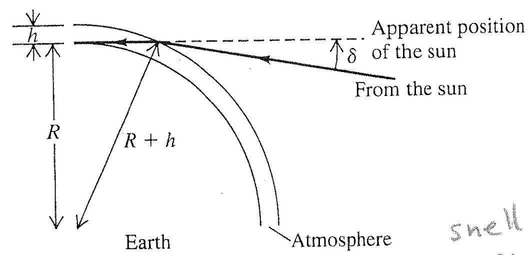


Figure 33.44 Problem 33.49.

$$\sin\alpha = \frac{R}{R+h}$$

Snell's Law $\Longrightarrow$ $\sin(\alpha+\delta) = n\sin\alpha$

$$\Rightarrow \alpha + \delta = \sin^{-1}\left[n\sin\alpha\right]$$

$$\Rightarrow \delta = \sin^{-1}\left[n\sin\alpha\right] - \alpha = \sin^{-1}\left[n\frac{R}{R+h}\right] - \sin^{-1}\left(\frac{R}{R+h}\right)$$

QED

b)

$$\delta = \sin^{-1}\left[\frac{(1.0003)\ 6.38\times10^6}{6.38\times10^6 + 2\times10^4}\right] - \sin^{-1}\left(\frac{6.38\times10^6}{6.38\times10^6 + 2\times10^4}\right)$$

$$= 85.6915^\circ - (85.469)^\circ \simeq \boxed{.22^\circ}$$