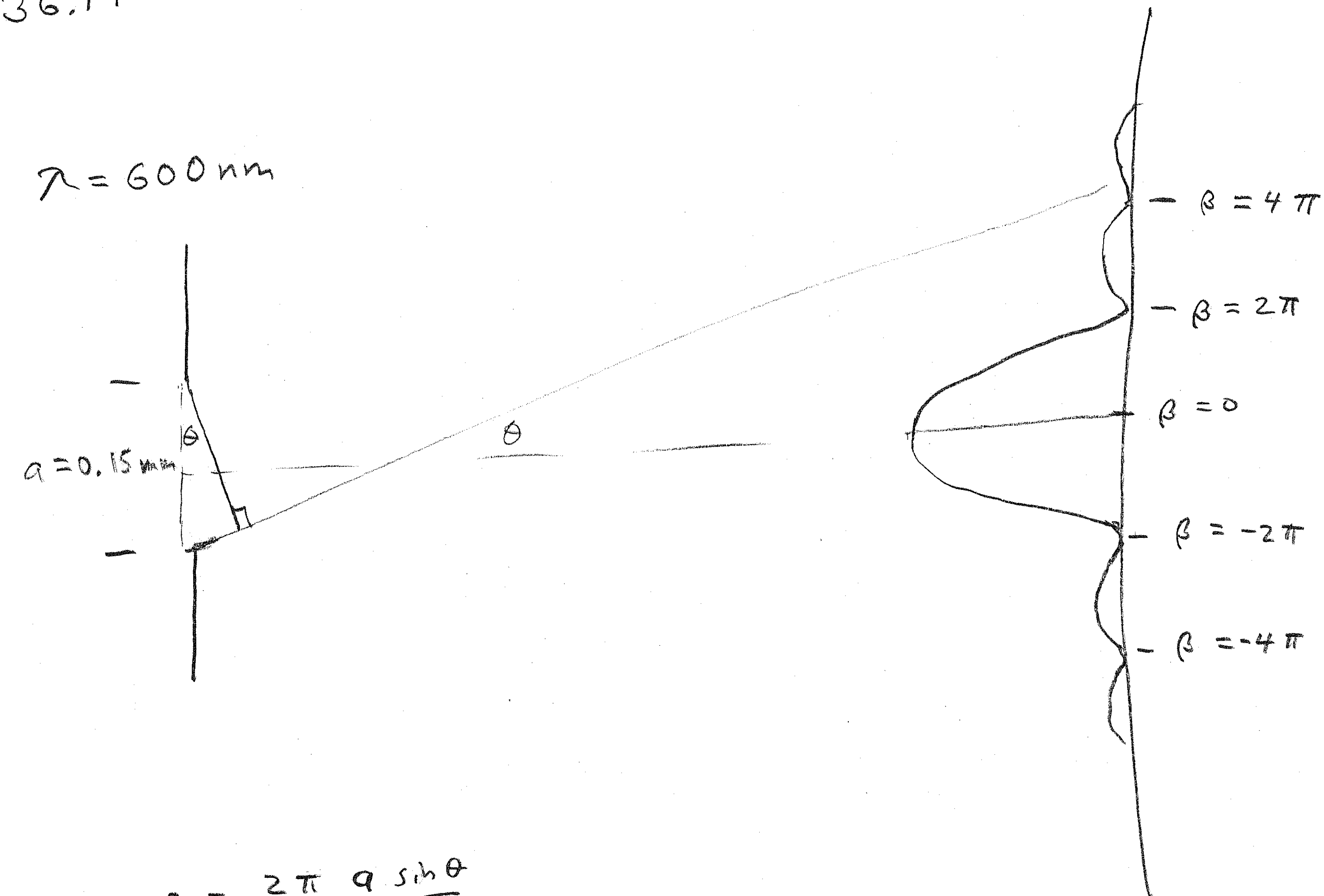
HW 29

From Text

36.14

$\lambda = 600\,\text{nm}$



$$\beta = \frac{2\pi\, a \sin\theta}{\lambda}$$

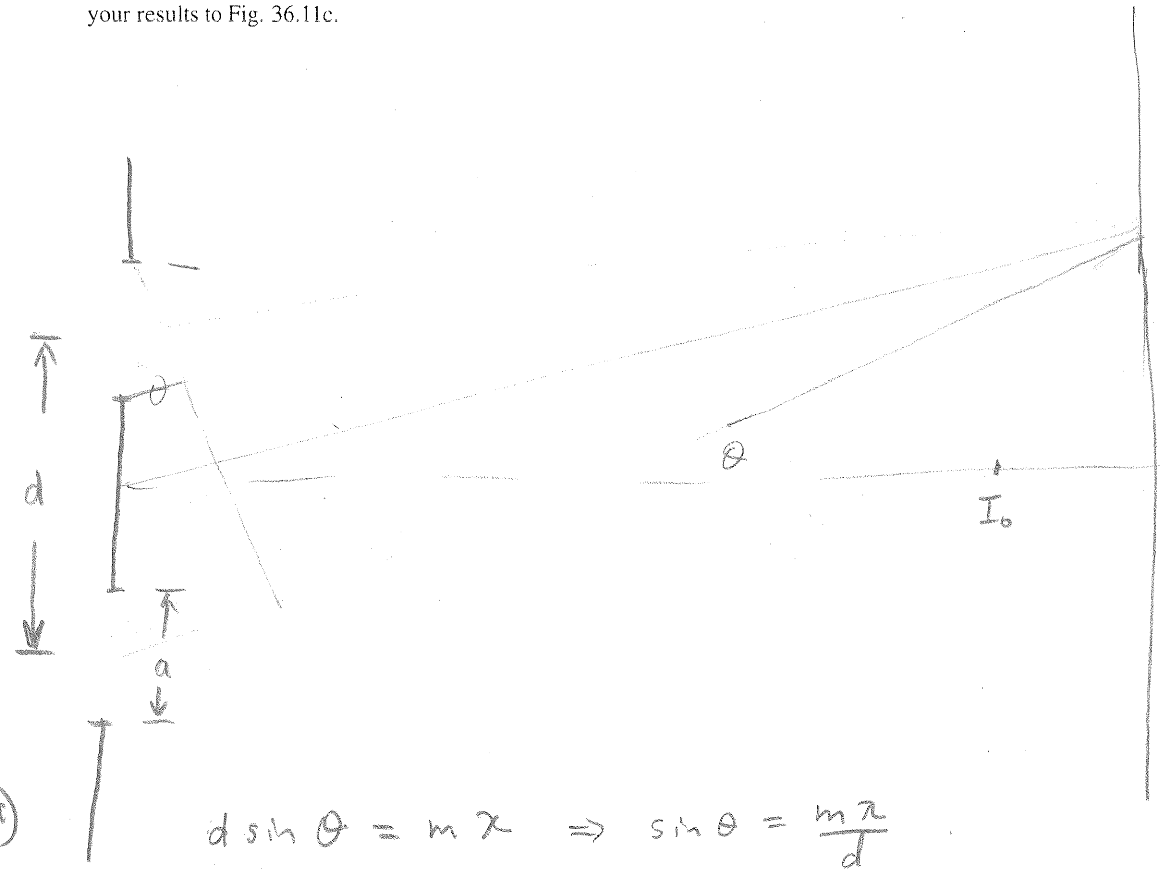
a) $\theta = 0$ $\quad \beta = \boxed{0}$

b) $\beta = \boxed{4\pi}$

c) $\theta = 7.0^\circ$ $\quad \beta = \dfrac{2\pi \; 0.15 \times 10^{-3}\,\text{m} \; \sin 7^\circ}{600 \times 10^{-9}\,\text{m}}$

$\cong \boxed{191.432 \text{ rad}} \simeq 61\,\pi$

36.18 Diffraction in an Interference Pattern. Consider the interference pattern produced by two parallel slits with width a and separation d. Let $d = 4a$. a) Ignoring diffraction effects due to the slit width, at what angles θ from the central maximum will the next five maxima in the two-slit interference pattern occur? (Your answer will be in terms of d and the wavelength λ of the light.) b) Now include the effects of diffraction. If the intensity at $\theta = 0$ is I_0, what is the intensity at each of the angles calculated in part (a)? Compare your results to Fig. 36.11c.



a) $d \sin\theta = m\lambda \Rightarrow \sin\theta = \frac{m\lambda}{d}$

$$\Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) \qquad m = 1, 2, 3, 4, 5}$$

b) $I = I_0\left[\frac{\sin \beta/2}{\beta/2}\right]^2 \qquad \beta = \frac{a \sin\theta}{\lambda} 2\pi \qquad \sin\theta = \frac{m\lambda}{d}$

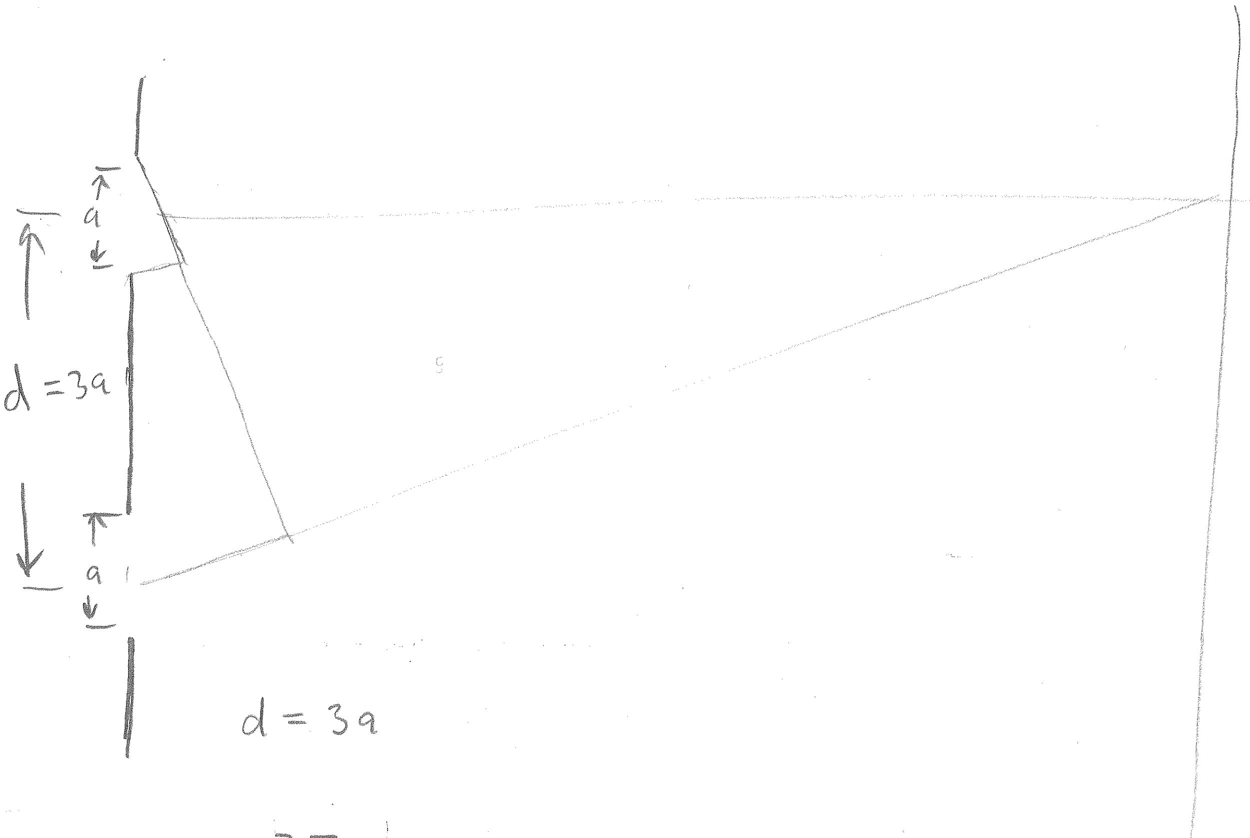
$$\Rightarrow \beta = m\frac{a}{d}2\pi = m\frac{a}{4a}2\pi = m\frac{\pi}{2}$$

$$I = I_0\left[\frac{\sin\left(m\frac{\pi}{4}\right)}{m\frac{\pi}{4}}\right]^2 \qquad m = 1, 2, 3, 4, 5$$

I $\Rightarrow I = \left(\frac{4}{\pi}\frac{1}{\sqrt{2}}\right)^2 I_0 = 0.811 I_0,\ 0.405 I_0,\ 0.0901 I_0,\ 0,\ 0.0324.$

m = 1, 2, 3, 4, 5

36.24 An interference pattern is produced by two identical parallel slits of width a and separation (between centers) $d = 3a$. Which interference maxima m_i will be missing in the pattern?



$d = 3a$

$\beta = \frac{2\pi}{\lambda} a \sin\theta = m\, 2\pi$ for diffraction zero

$m \neq 0 \qquad m = 1, 2, 3,$

$d \sin\theta = n\lambda$ for a maximum in interference

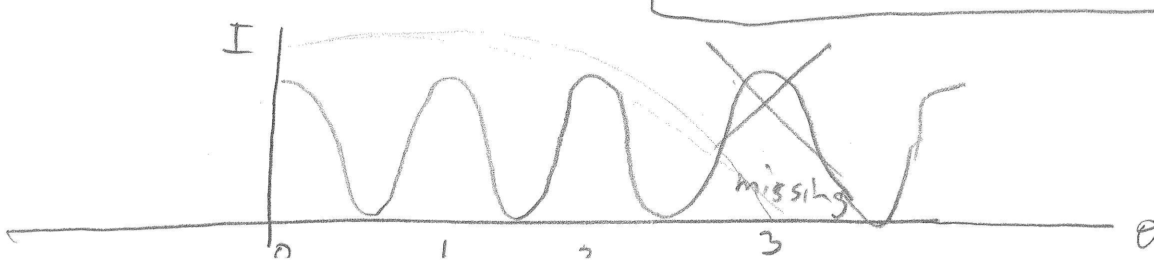
$n = 0, 1, 2, 3, \ldots$

$\Rightarrow$ $3a \sin\theta_{max} = n\lambda$ & $a \sin\theta_{min} = m\lambda$

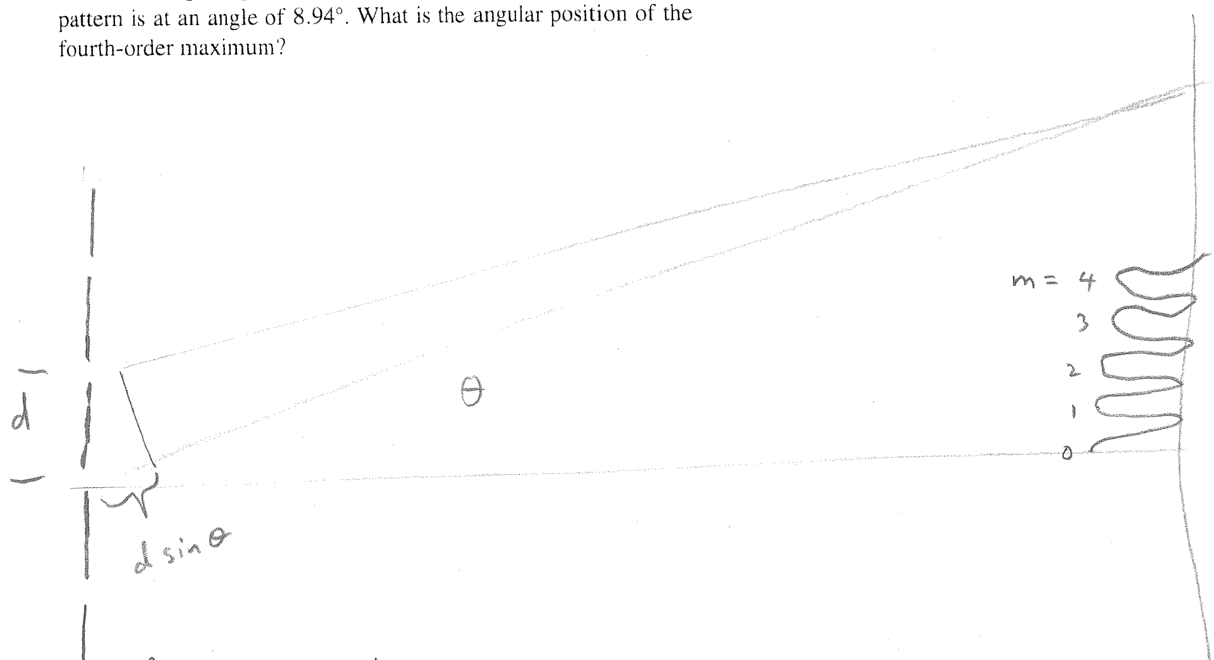
$\Rightarrow a \sin\theta_{max} = \frac{n}{3}\lambda$ & $a \sin\theta_{min} = m\lambda$

$\theta_{max} = \theta_{min} \Rightarrow \frac{n}{3} = m \Rightarrow n = 3m$

$\boxed{n = 3, 6, 9 \text{ will be missing}}$



36.28 Monochromatic light is at normal incidence on a plane transmission grating. The first-order maximum in the interference pattern is at an angle of 8.94°. What is the angular position of the fourth-order maximum?



for a maximum

$$d \sin\theta_m = m\lambda \qquad \theta_1 = 8.94^\circ$$

$$\frac{\sin\theta_m}{\sin\theta_n} = \frac{m}{n} \Rightarrow \sin\theta_n = \frac{n}{m}\sin\theta_m$$

$$\Rightarrow \theta_4 = \sin^{-1}\left[\frac{4}{1}\sin(8.94^\circ)\right] = \boxed{38.43^\circ}$$

36.39 Due to blurring caused by atmospheric distortion, the best resolution that can be obtained by a normal, earth-based, visible-light telescope is about 0.3 arcsecond (there are 60 arcminutes in a degree and 60 arcseconds in an arcminute). a) Using Rayleigh's criterion, calculate the diameter of an earth-based telescope that gives this resolution with 550-nm light. b) Increasing the telescope diameter beyond the value found in part (a) will increase the light-gathering power of the telescope, allowing more distant and dimmer astronomical objects to be studied, but will not improve the resolution. In what ways are the Keck telescopes (each of 10-m diameter) atop Mauna Kea in Hawaii superior to the Hale Telescope (5-m diameter) on Palomar Mountain in California? In what ways are they *not* superior? Explain.

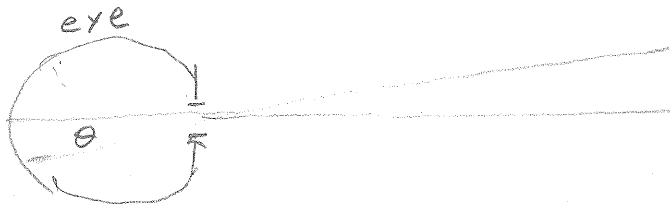
a) $$D = \frac{1.22\lambda}{\sin\theta} \simeq \frac{1.22\lambda}{\theta} = \frac{1.22\,(550\,\text{nm})}{(0.3)\,\text{arc sec}\ \frac{\text{deg}}{3600\ \text{arc sec}}\ \frac{2\pi\,\text{rad}}{360\,\text{deg}}}$$

$$= 4.61 \times 10^{8}\ \text{nm} = \boxed{0.461\ \text{m}}$$

b) Keck telescope can look at fainter objects because D is Larger.

The Hale may have greater resolution because it is at higher ground and there will be less atmospheric distrortion then at Mauna Kea.

36.40 If you can read the bottom row of your doctor's eye chart, your eye has a resolving power of one arcminute, equal to $\frac{1}{60}$ degree. If this resolving power is diffraction-limited, to what effective diameter of your eye's optical system does this correspond? Use Rayleigh's criterion and assume $\lambda = 550$ nm.

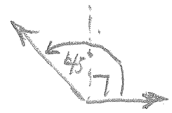


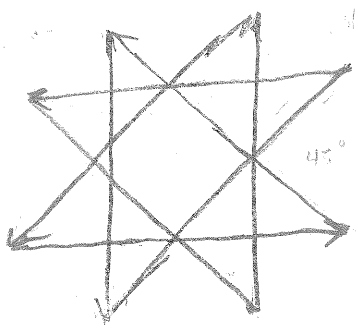
$$D = \frac{1.22\,\lambda}{\sin\theta_1} = \frac{1.22\,\lambda}{\theta_1} = \frac{(1.22)\,550\times10^{-9}\ \text{m}}{\frac{1}{60}\ \text{deg}\ \frac{\pi\ \text{rad}}{180\ \text{deg}}}$$

$$= 2.3\times10^{-3}\ \text{m} = \boxed{2.3\ \text{mm}}$$

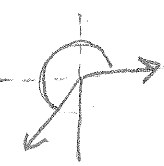
36.59 Phasor Diagram for Eight Slits. An interference pattern is produced by eight equally spaced, narrow slits. Figure 36.13 shows phasor diagrams for the cases in which the phase difference ϕ between light from adjacent slits is $\phi = \pi$, $\phi = \pi/4$, and $\phi = \pi/2$. Each of these cases gives an intensity minimum. The caption for Fig. 36.13 also claims that a minimum occurs for $\phi = 3\pi/4$, $\phi = 5\pi/4$, $\phi = 3\pi/2$, and $\phi = 7\pi/4$. a) Draw the phasor diagram for each of these four cases, and explain why each diagram proves that there is in fact a minimum. (*Note:* You may find it helpful to use a different colored pencil for each slit!) b) For each of the four cases $\phi = 3\pi/4$, $\phi = 5\pi/4$, $\phi = 3\pi/2$, and $\phi = 7\pi/4$, for which pairs of slits is there totally destructive interference?

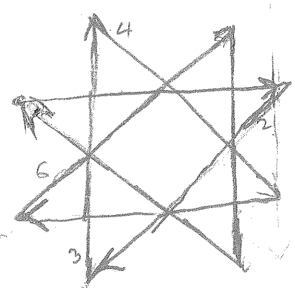
a) $\phi = \frac{3}{4}\pi = \frac{3}{8}(2\pi) =$



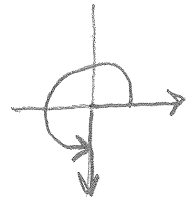


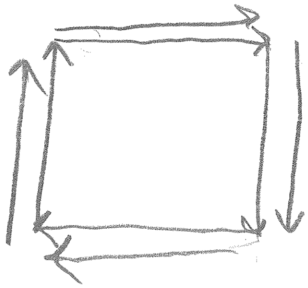
$\phi = \frac{5}{4}\pi = \frac{5}{8}\,2\pi$



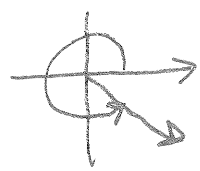


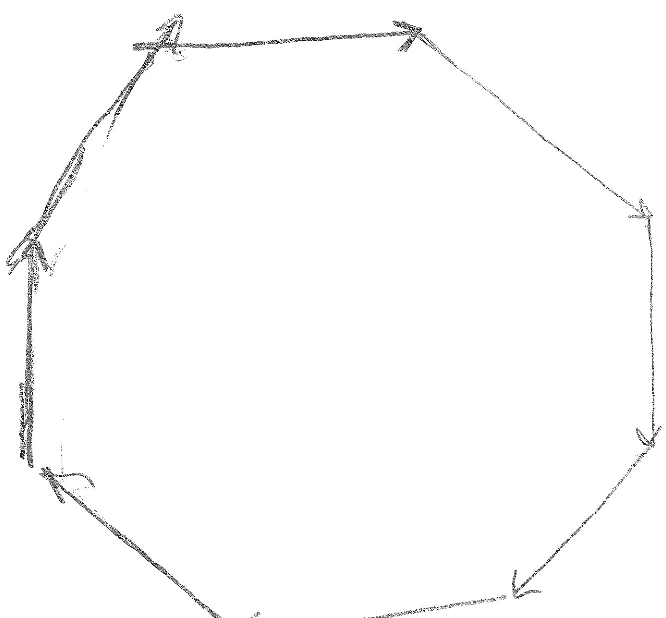
$\phi = \frac{3}{2}\pi = \frac{3}{4}2\pi$





$\phi = \frac{7}{4}\pi = \frac{7}{8}(2\pi)$





b) $\phi = \frac{3\pi}{4}$, $\frac{5}{4}\pi$, $\frac{3}{2}\pi$, $\frac{7}{4}\pi$

slits apart 8 , 8 , 4 , 8 for distructive interference