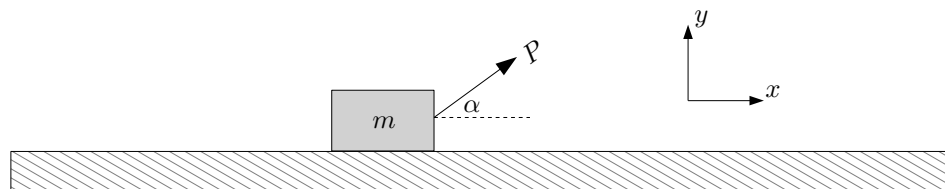Box your answers.

1 [15pts] Pulling a Block

A block, of mass m, is pulled along a flat level plane, by a string that is attached to the block. The string pulls up at an angle α from the plane, with a constant force with a magnitude of P. The coefficient of kinetic fiction between the block and the plane is μ_k. The usual gravitational field acts down, $\vec{g} = -g\,\hat{y}$.

1.1 (5) Free Body Diagram

Draw a free body diagram of the block.

1.2 (10) Acceleration

Find the acceleration, $\ddot{x}$, of the block.

2 [10pts] Moment of Inertia

Find the moment-of-inertia tensor, $\mathbf{I}$, for rotations about the origin for one particle, with mass m, that is located on the x axis at the position $(x, y, z) = (a, 0, 0)$. Write your answer in the form

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} . \tag{2.1}$$

3 [20pts] Inverse Rocket (Again)

A large abandoned space ship travels through space which is filled with uniformly distributed "space dust," with mass density ρ. The only forces on the space ship are from the dust that collects on the ship as it goes through the dust. Consider the space dust to be at rest (not moving) before the ship hits it. Assume that all the dust that gets hit by the ship sticks to the ship and effectively increases the mass of the ship, and slows down the ship. All the motion is in one dimension.

3.1 (10) Find the Rate of Change of Mass of the Ship

A is the cross-sectional area of the ship that is passing (cutting) through the dust. Let m be the mass of the ship and the collected dust. Let v be the speed of the ship and the collected dust. Find rate of change of mass of the ship and the collected dust, $\frac{dm}{dt}$, as a function v, ρ, and A. Use the fact that the ship gains the mass of all the dust that it hits.

3.2 (10) Power

Find the rate at which kinetic energy is lost, P, as a function of v, ρ and A. In this case this power is the rate at which heat is generated on the front of the ship as the dust inelastically collides with it.

4 [15pts] Tether-ball
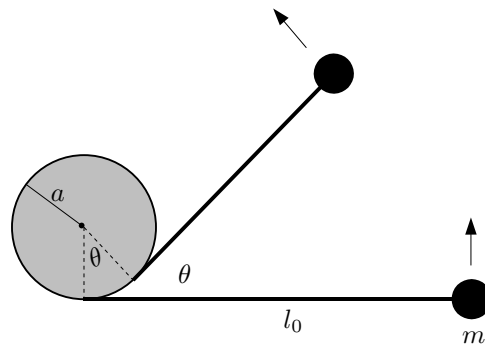
A tether-ball post has a radius a. A small ball with mass m swings on the end of the massless stretch-less rope. The length of the rope decreases as the angle θ increases, as the ball gets closer to the pole. Consider all the motion to be in the horizontal plane, so the effect of gravity is ignored. Think of it as a tether-ball on a smooth floor. The length of the rope when θ is zero is l_0.
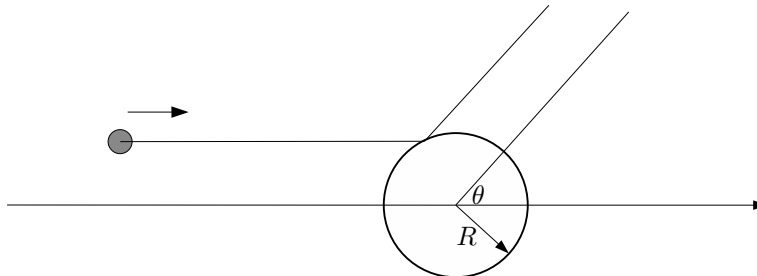
4.1 (10) Lagrangian

Find the Lagrangian, $L(\theta, \dot{\theta})$, for the tether-ball system using θ as your generalized coordinate variable.

4.2 (5) Equation of Motion

Find the second order ordinary differential equation for the motion of θ.

5 [20pts] Two Dimensional Hard Sphere Scattering

We will look at the analog of hard smooth sphere scattering, but with motion in just two dimensions. Consider all motion to be constrained to a plane, kind of like hockey pucks on an ice rink. We have smooth circular target particles with radius R. The target particles are fixed. The projectiles are small and scatter elastically off the target particles. There are N_{tar} sparsely distributed target particles over a length L along a direction that is perpendicular to a beam of small projectiles with a uniform intensity of I_{inc} (particles per unit length, per unit time). The width of the beam of particles is much larger than R, and smaller than all the target particles together (the whole target), as our scattering theory requires.

5.1 (10) Particles Not Scattered

Find the intensity (particles per unit length, per unit time) of the beam of small projectiles that go through the target undeflected, I_{un}.

5.2 (10) Differential Cross Section

The width of the incident beam of particles is W ($W < L$ and $W \gg R$). Find the intensity (particles per unit length, per unit time) of the beam of small projectiles that is scattered at an angle θ at a distance D from the target ($D \gg L \gg R$), I_{sc}. In this case $-\pi \leq \theta \leq \pi$ and we don't need to consider solid angle. Hint: Start by finding the differential number of scatters, $\mathrm{d}N_{\text{sc}}$, as a function of the differential impact parameter, $\mathrm{d}b$, and then consider the impact parameter, b, to be a function of θ.

6 [20pts] Circular Slide
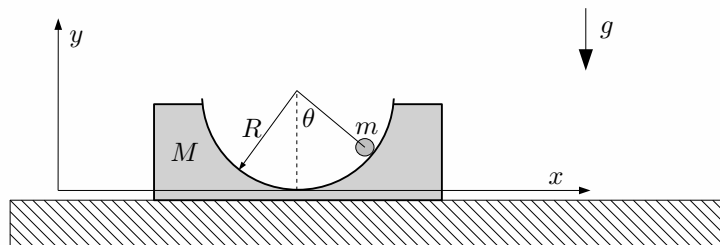
The figure above shows a particle with mass m that slides on a frictionless circular slide. The circular slide has a radius R. The circular slide is cut from a block of that slides without friction on a flat table. The mass of the circular slide block is M. There is a uniform gravitational field with magnitude g.

6.1 (10) Lagrangian

We define the x position of the center-of-mass, x_{cm}, as

$$x_{\mathrm{cm}} \equiv \frac{m\,x_m + M\,x_M}{m + M} \tag{6.1}$$

where x_m is the x position of the particle and x_M is the x position of the center-of-mass of the block. Find the Lagrangian of this system using the generalized coordinate variables x_{cm} and θ, $L(\dot{x}_{\mathrm{cm}}, \theta, \dot{\theta})$. Hint: Start by writing T as a function of the speeds of the two objects separately, keeping in mind that the particle moves in circular motion relative to the block.

6.2 (5) Differential Equation of Motion

From your Lagrangian, find the differential equations of motion for x_{cm} and θ.

6.3 (5) Find the Angular Frequency of Small Oscillations

Find the angular frequency for small θ oscillations, ω_0. Hint: $\theta\dot{\theta}^2 \approx 0$.