1 [20pts] Gravitation

A uniform solid sphere of mass M and a radius R is fixed a distance h from it's center above a thin infinitely long cylindrical mass with uniform mass density λ (mass/length). h is greater than R. What is the force on the sphere from the cylindrical mass? Using Gauss's law is the easiest way to do this problem, but it is not required. Anyone doing this the hard way may need

$$\int \frac{\mathrm{d}x}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}} + C \qquad \text{or} \qquad \int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \tag{1.1}$$

2 [20pts] Stationary Integral

Find $y(x)$ such that the following integral is stationary,

$$J = \int_{x_1}^{x_2} \left(y'^2 - y^2\right) \mathrm{d}x, \tag{2.1}$$

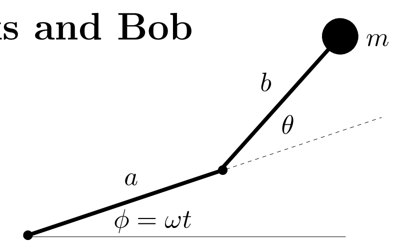
where $y' \equiv \frac{\mathrm{d}y}{\mathrm{d}x}$. You may need

$$\int \frac{\mathrm{d}u}{\sqrt{a^2 - u^2}} = \cos^{-1}\frac{u}{|a|} + C\,, \quad a^2 > u^2\,. \tag{2.2}$$

3 [30pts] Tension in a Simple Pendulum

We have a simple pendulum with length l, bob mass m, and in a uniform gravitational field g. Use Lagrangian dynamics with an undetermined multiplier, λ, and Lagrangian $L\left(r, \theta, \dot{r}, \dot{\theta}\right)$, where θ is the angle from the vertical, and r is the length of the pendulum, to find: **(a)** an equation of constraint, f, **(b)** the Lagrangian, **(c)** the equations of motion for θ and r, **(d)** λ, and **(e)** the tension, T, in the pendulum. All may be a function of r, $\dot{r}$, θ, $\dot{\theta}$, m, g, and l.

4 [30pts] Swinging Sticks and Bob



The two massless sticks are connected together at one end and they of free to pivot in a plane about that end. The other end of one of the sticks has a bob of mass m fixed to it. The other end of the other stick is fixed at a point, which it rotates about at a constant angular speed of ω. The stick without the bob on an end has a length a and the stick with the bob on the end has a length of b ($a \neq b$). All of the motion is in a plane. There is no gravity acting on the bob. Use θ as the generalized coordinate variable, as shown in the figure above. (Pretty much what you see in the figure above.)

4.1 (15pts) Lagrangian

Find the Lagrangian, $L(\theta, \dot{\theta})$ for this system. Note that $U = 0$.

4.2 (10pts) θ Equation of Motion

Find the equation of motion for θ ($\ddot{\theta} =?$).

4.3 (5pts) Equilibrium and Stability

Find the equilibrium θ positions θ_0. Determine if these positions are stable or not. For any stable positions, find the angular frequency of oscillation about that equilibrium position.