1 how small is displacement to make simple harmonic motion

For a large class of forces we can expand the force function into a Taylor series. For example springs have forces that behave like $F = -kx$, where k is a constant and x is the elongation in the spring. This works so long as the value of x is limited in size, but for larger values of x the springs may behave more like $F = -kx - k_3x^3$.

A particle that is constrained to move in one dimension has following net force

$$F(x) = -k_1\,x - k_2\,x^2 - k_3\,x^3, \tag{1.1}$$

where k_1, k_2, and k_3 are positive constants, and x is the displacement of the particle. It is easy to see that $x = 0$ is an equilibrium position, since $F(0) = 0$. And so this is a pretty general form for a 1-D force expanded near equilibrium. (We are excluding discontinuous functions.) No matter what the values of k_1, k_2, and k_3 have we can always have a small enough value of x such that the linear force term is dominant.

How small must x be such that the size of the linear force $-k_1x$ term is 100 or more times the quadratic force term $-k_2x^2$? Also, how small must x be such that the size of the linear force term $-k_1x$ is 100 or more times the cubic force term $-k_3x^3$? Express your answers in terms of k_1, k_2, and k_3.

2 energy

A simple harmonic oscillator consisting of a block and a spring has an frequency $f_0 = 2$Hz. There is no friction or driving force. The block has a mass $m = 100g$. The block is set into motion, with an initial speed of 10 cm/s, from its equilibrium position. Find the maximum potential energy in the spring, $U_{\max}$, and the amplitude, A, of the oscillation. Assume that the potential energy is zero at the equilibrium position. Ignore gravity.