1 Stationary Integral

Find the differential equations that $x(t)$ and $y(t)$ must satisfy such that the following integral is stationary,

$$J = \int_{t_1}^{t_2} \left(\frac{1}{2}\,\dot{x}^2 + \frac{1}{2}\,\dot{y}^2 - k\,xy + xA\cos\omega t \right) \mathrm{d}t, \tag{1.1}$$

where $\dot{x} \equiv \frac{\mathrm{d}x}{\mathrm{d}t}$, $\dot{y} \equiv \frac{\mathrm{d}y}{\mathrm{d}t}$, and k, A, and ω are constants. Hints: Use Euler's equations. You do not need to solve for $x(t)$ and $y(t)$, just find the differential equations that $x(t)$ and $y(t)$ must satisfy.