1 Shortest Path in Between Two Points

Show that the shortest path between two points in three-dimensional space is that of a straight line. Hint: Use Euler's equations with several dependent variables $x(t)$, $y(t)$, and $z(t)$, with t being the independent variable.

2 Stationary Integral

Find $y(x)$ such that the following integral is stationary,

$$J = \int_{x_1}^{x_2} \sqrt{x}\sqrt{1+y'^2}\,\mathrm{d}x, \tag{2.1}$$

where $y' \equiv \frac{\mathrm{d}y}{\mathrm{d}x}$.

3 Geodesic: Shortest path on a Circular Cylinder, Three Ways

3.1 Using One Dependent Variable $z(\phi)$

Show that the shortest path between two points on a circular cylinder is along a helix. Do this with the assumption that, $z(\phi)$ may be written as a function of ϕ in cylindrical coordinates (r, ϕ, z) with r being a constant. The equation of a helix in cylindrical coordinates is $z = c_1\phi + c_2$ where c_1 and c_2 are constants.

3.2 With Two Dependent Variables

Show that the shortest path between two points on a circular cylinder is along a helix. Do this using cylindrical coordinates with the variables $(r, \phi(t), z(t))$. That is $\phi(t)$, and $z(t)$ are dependent on an independent variable t, and r being a constant. Note: You do not have to solve the t dependence of ϕ and z.

3.3 With an Equation of Constraint and Three Dependent Variables

Show that the shortest path between two points on a circular cylinder is along a helix. Do this using cylindrical coordinates with the variables $(r(t), \phi(t), z(t))$ all dependent on an independent variable t, and with the constraint that $r = a$, where a is a constant. Note: There's not much different between this and the last sub-problem.

4 Minimum Area of Revolution

A surface is generated by revolving a curve $y(x)$, that connects two points in the x-y plane, around the x axis. Find this curve such that the surface area is a minimum. Consider using the second form of Euler's equation.

5 Shape of a Hanging Cable

A cable with uniform linear mass density, and a fixed length l is hung across space while gravity pulls down on it causing the cable to sag between the two points at the ends of the cable. The total gravitational potential energy (mgy) from the earth pulling down on the cable is a minimum. What is the general shape of the cable, $y(x)$. Hint: Use the method on page 222 of Thorton and Marion.