1 Pendulum with Vertically Moving Pivot

A frictionless pendulum swings in the x-y plane. The x direction is down in the direction of the uniform gravitational field g. The y direction is to the right. The pendulum bob has a mass of m. The pendulum has a length l. The position of the pivot point of the pendulum is moving (up and down) along the x-direction with the pivot position given by $x_s(t)$.

1.1 Lagrangian

Find the Lagrangian for this system $L(\theta, \dot{\theta}, t)$ where θ is the angle that the pendulum makes with the (vertical) x axis toward the y axis. Answer in terms of m, g, l, $x_s(t)$, $\dot{x}_s(t)$, θ, and $\dot{\theta}$. You may consider $x_s(t)$ and $\dot{x}_s(t)$ as given functions of t, so $L(\theta, \dot{\theta}, t) = L(\theta, \dot{\theta}, x_s(t), \dot{x}_s(t))$

1.2 Equations of Motion

Using this Lagrangian find the equations of motion for θ (something like $\ddot{\theta} =?$). Answer in terms of m, g, l, $x_s(t)$, $\dot{x}_s(t)$, $\ddot{x}_s(t)$, θ, $\dot{\theta}$, and $\ddot{\theta}$.