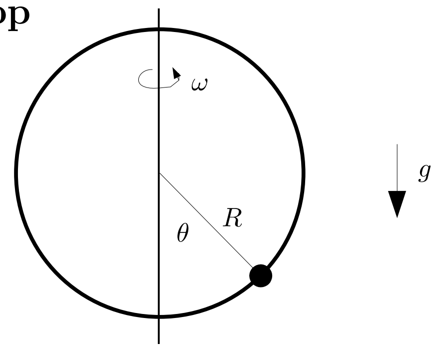
1 Bead on a Spinning Hoop



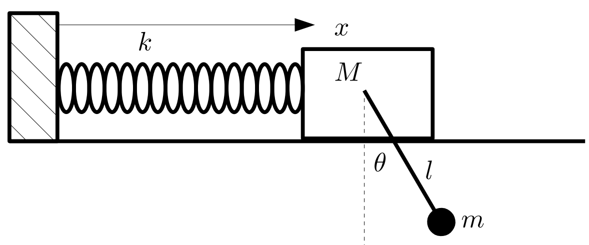
A bead of mass m is constrained to move, without friction, along a circular wire hoop of radius R. The wire hoop spins at a constant angular speed, ω, about a vertical axis through its center. A uniform gravitational field, g, acts on the bead. Let θ be the angular position of the bead as measured from a vertical line to a line from the center of the hoop to the bead, as shown in the figure above.

a) Find the equation of motion of θ ($\ddot{\theta} =?$),

b) the three equilibrium positions of θ (call them θ_0), and

c) determine the conditions for the stability of these equilibrium positions.

2 Spring and Block, and Pendulum



A spring, with spring constant k, is connected to a fixed wall at one end, and a block, of mass M, that slides without friction, at the other end. The spring has a rest length of a. A simple pendulum with length l and bob mass m is pivoted on the sliding block. The simple pendulum moves in the plane of the motion of the block and spring. A uniform gravitational field, g, acts down on the pendulum bob. Let x be the position of the sliding block and θ be the angle of the pendulum measured from the vertical. Find the Lagrangian, $L(x, \theta, \dot{x}, \dot{\theta})$, and the equations of motion of x and θ ($\ddot{x} =?$, $\ddot{\theta} =?$).

3 Spherical Pendulum

A spherical pendulum is a pendulum that is free to swing in both sideways directions. By contrast a simple pendulum swings in a fixed vertical plane. A good choice of coordinates, for describing the motion of a spherical pendulum, is spherical polar coordinates (r, θ, ϕ) with r unit vector pointing straight down when $\theta = 0$ (positive z is down). The two variables θ and ϕ can be the two generalized coordinates that completely describe the motion of the spherical pendulum. We have a spherical pendulum with a bob mass m and length l.

3.1 Lagrangian and Equations of Motion

Find the Lagrangian and the two coupled equations of motion of this spherical pendulum ($L(\theta, \phi, \dot{\theta}, \dot{\phi})$), $\ddot{\theta} =?$, and the ϕ equation of motion).

3.2 Angular Momentum Along z Axis

Explain what the ϕ equation says about the angular momentum in the z direction, l_z. Also replace the $\dot{\phi}$ in the θ equation with a function of θ, l_z, and other constants.

3.3 Equilibrium θ Positions

Finding the equilibrium θ positions requires solving a small, but non-trivial transcendental equation. Though there is a closed form solution, computing it is not required for this part of the problem.

a) Find range of values of the equilibrium θ position, θ_0, for all values of l_z $(0 \leq l_z^2 \leq \infty)$,

b) explain how to find the equilibrium θ position, θ_0, in general (there is more than one correct answer), and

c) describe the motion of the pendulum near this equilibrium θ position, θ_0, including a determination of the stability of the motion near θ_0.