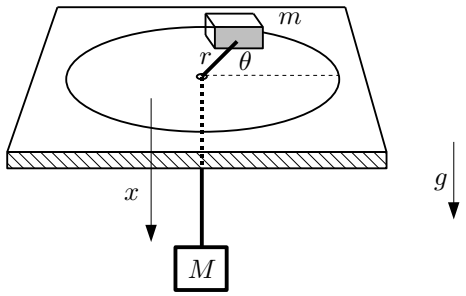
1 Spinning Block and Hanging Weight using Lagrange Multiplier



A block, with mass m, slides without friction on a level table. A stretch-less, massless string connects this block to a hanging weight, with mass M, through a hole in the table. The block on the table can orbit around the table so that the angle θ and the radial distance r both change with time. When r changes the distance from the table to the hanging weight, x, changes too. This gives us the constraint equation $f = r + x - a = 0$, where a is a constant. Consider your dynamical (generalized coordinate) variables to be θ, r, and x.

1.1 Find Equations of Motion

Find the equations of motion for θ, r, and x using a Lagrange multiplier λ, with the given constraint equation. You do not have to solve them, or solve for λ.

1.2 Remove the $\dot{\theta}$ Dependence in the r Equation

The integration constant for the θ equation is the angular momentum of the block about the hole, l. Find the first integral of the θ equation and remove the $\dot{\theta}$ in the r equation of motion.

1.3 Lagrange Multiplier, λ

Physically what is the Lagrange multiplier, λ? In general, is it constant or a function of time? Justify your answer.