# 1 Sliding Car

A car traveling down an incline with a 8% grade (raise/run) locks his brakes and skids 30 m before hitting a parked car. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.45$. Was the car exceeding the 25 MPH speed limit? Explain.

# 2 Grandfather Clock

A grandfather clock has a pendulum length of 0.7 m and a bob mass of 0.4 kg. A weight of mass 2 kg falls 0.8 m in seven days to keep the amplitude (from equilibrium) of the pendulum oscillating steady at 0.03 rad. What is the quality factor, $\mathcal{Q}$, of this clock? Assume that all the energy is lost in the oscillating pendulum.

# 3 Gravitation

A uniform solid sphere of mass $M$ and a radius $R$ is fixed a distance $h$ above a thin infinite sheet of mass density $\rho_s$ (mass/area). $h$ is greater than $R$. What is the force on the sheet from the sphere?

# 4 A Particle in a Cone

A particle, with mass $m$, is constrained to move on the surface of a cone. The cone has it's vertex pointing down in the direction of gravity ($g$). The cone has a half-angle $\alpha$.

## 4.1 Lagrangian

Write the Lagrangian, $L\left(r, \phi, \dot{r}, \dot{\phi}\right)$, in terms of spherical polar coordinates $r$, and $\phi$, where the $\theta$ coordinate is fixed at value $\alpha$ on the surface of the cone.

## 4.2 Equations of Motion

Find the equations of motion for $r$ and $\phi$. Interpret the $\phi$ equation in terms of the angular momentum along the $z$ direction, $l_z$. Use $l_z$ to eliminate the $\dot{\phi}$ from the $r$ equation of motion.

## 4.3 Find an Equilibrium $r$ Position

Find the equilibrium $r$ position, $r_0$. Determine if this equilibrium $r$ position is stable or not. If this position is stable, find the frequency of oscillation about this equilibrium position.

# 5 Non-unique Lagrangian

Show that the if a Lagrangian $L\left(q_1, ..., q_s, \dot{q}_1 ..., \dot{q}_s, t\right)$ is related to another Lagrangian $L'\left(q_1, ..., q_s, \dot{q}_1 ..., \dot{q}_s, t\right)$ by $L' = L + \frac{\mathrm{d}F}{\mathrm{d}t}$, where $F = F\left(q_1, ..., q_s, t\right)$, then the two Lagrangians will give exactly the same equations of motion.