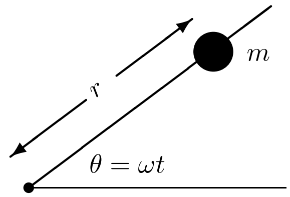
1 Bead on a Spinning Wire

This walks you through the problem.



A bead, with mass m, slides without friction on a long straight wire. The wire is spinning about one end at a constant angular speed ω. Let r be the position of the bead on the wire as measured from the point of rotation of the wire. There is no gravity. All the motion is in a plane.

1.1 Lagrangian

In terms of r, $\dot{r}$, m, ω, and t, write an expression for the Lagrangian $L(r, \dot{r}, t)$ for this system. Note that $U = 0$.

1.2 Lagrange's Equation of Motion

Using your Lagrangian, find the equation of motion for r.

1.3 Canonical Conjugate Momentum

In general, the canonical momentum that is conjugate (paired) to generalized position q_i is $p_{q_i} \equiv p_i = \frac{\partial L}{\partial \dot{q}_i}$.

The canonical momentum that is conjugate to r is $p_r \equiv \frac{\partial L}{\partial \dot{r}}$. Find the canonical momentum that is conjugate to r as a function of r, $\dot{r}$, ω, and t.

1.4 Hamiltonian

The Hamiltonian in general can be defined as

$$H(q_k, p_k, t) = \sum_j p_j \dot{q}_j - L(q_k, \dot{q}_k, t) \ , \tag{1.1}$$

where the Hamiltonian must be written as a function of just the dependent dynamical variables q_k, p_k, and t. There must be no $\dot{q}_k$ dependence in H. So we are assuming that $\dot{q}_k$ can be written as a function of q_k, p_k, and t, in order to make this transformation.

In our case $H(r, p_r) = p_r \dot{r} - L(r, \dot{r})$. In terms of r, p_r, m, and ω, find the Hamiltonian, $H(r, p_r, t)$ for this system.

1.5 Hamilton's Equations of Motion

In Lagrangian dynamics, for each generalized position, q_i, $(i = 1, 2, ...s)$ there is an equation of motion that is a second-order ordinary differential equation. In Hamiltonian Dynamics, for each corresponding second-order differential equation of Lagrangian dynamics there are two first-order ordinary differential equations, one of the generalized position, q_i, and one for the corresponding canonical momentum, $p_{q_i} \equiv p_i$. These Hamiltonian equations of motion are gotten from the Hamiltonian like so

$$\dot{q}_i = \frac{\partial H}{\partial p_{q_i}} \quad \text{and} \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{1.2}$$

Use the Hamiltonian that you just calculated and equations 1.2 to find the equations of motion for r and p_r, and show that these equations of motion for r and p_r are equivalent to the equation of motion from section 1.2.

1.6 Energy vs Hamiltonian

In terms of r, p_r, m, ω, and t, **(a)** find the difference between the total energy E and the Hamiltonian H for this system $(E - H)$. **(b)** Is H constant in time? **(c)** Is E constant in time? Note that $E = T = L$. Explain you results.