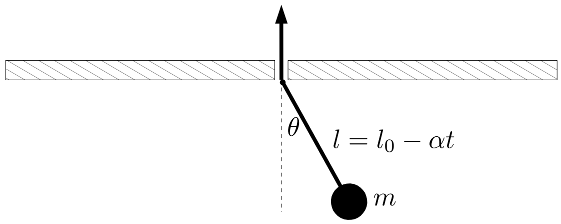
1 Shrinking Pendulum



A simple plane pendulum consists of a mass m attached to a massless string of length l. The suspension point of the pendulum remains fixed as the length, l, is shortened at a constant rate α, as shown above. The pendulum has a length l_0 at time $t = 0$. In this problem we'll find the equations of motion for the pendulum using the Lagrangian and the Hamiltonian method.

1.1 Lagrangian

In terms of θ $\dot{\theta}$, m, α, l_0, and t, write an expression for the Lagrangian, $L\left(\theta, \dot{\theta}, t\right)$, for this system.

1.2 Lagrange's Equations of Motion

Using your Lagrangian and Largrange's equatons, find the differential equation of motion for θ.

1.3 Canonical Conjugate Momentum

Find the canonical momentum that is conjugate (paired) to θ. That is, find $p_\theta \equiv \frac{\partial L}{\partial \dot{\theta}}$, as a function of θ $\dot{\theta}$, m, α, l_0, and t.

1.4 Hamiltonian

In terms of θ, p_θ, m, α, l_0, and t, write the Hamiltonian, $H(\theta, p_\theta, t)$ for this system.

1.5 Hamilton's Equations of Motion

Find the Hamilton's equations of motion for θ and p_θ, and show that these equations of motion for θ and p_θ are equivalent to the equations of motion from subsection 1.2.

1.6 Energy vs Hamiltonian

In terms of m, α, l_0, and t, find the difference between the total energy, $T + U$, and the Hamiltonian for this system.