

In this homework, you may use results from the text. Put your answers in terms of given quantities. Though the calculations here can be done as plug-ins, it's highly recommended that you do some deriving on your own to gain some understanding.

## 1 Scattering from Hard Spheres

A target consists of  $N_{\text{tar}}$  sparsely distributed smooth hard spheres of mass  $m_2$ . The radius of each these target spheres is  $R$ . A beam of small particles with mass  $m_1$  is incident on the target, where  $m_1 \ll m_2$ . The speed of the incident particles is  $u_1$ . The cross-sectional area of the entire target is  $A$ .  $A \gg R^2$ . The beam of small particles shoots a total of  $N_{\text{inc}}$  particles at the target, where  $N_{\text{inc}}$  is a large number. The cross-sectional area of the beam of small particles is much larger than  $R^2$ , but smaller than  $A$ . The small particles scatter elastically from the hard spheres.

### 1.1 Calculations

(a) Out of all the incident particles,  $N_{\text{inc}}$ , what is the number of  $m_1$  particles that are not scattered by any hard spheres (the number of  $m_1$  particles that go through the target with no deflection),  $N_{\text{un}}$ . (b) What is number of particles per unit area that go off at an angle of  $\theta$  measured from the direction of the incident beam, where  $\theta = 0$  would be no scattering (deflection), and at a distance of  $D$  from the target, where  $D^2 \gg A$ .

### 1.2 Conceptual Questions

This is an idealized scattering experiment. We have been a little more explicit in describing the relative parameter sizes than what is typically found in text book problems. There are a few approximations used in coming up with our simple scattering model. Without these simplifying assumptions our simple scattering model would not be the same.

Give **brief** answers to the following questions. Think about what would happen to our simple model in extreme cases. How would our scattering model be different or invalid if:

1. we had more hard spheres, such that they become densely distributed in the target,
2. the beam of  $m_1$  particles was not sparse, but dense in particles (high intensity),
3.  $N_{\text{inc}}$  was not a large number,
4. the cross-sectional area of the beam of small particles was not smaller than  $A$ ,
5. the cross-sectional area of the beam of small particles was smaller than  $R^2$ ,
6. the beam of small particles was a beam of larger particles of radius  $r$  which is close in size to  $R$ , the radius of the target spheres,
7. we were given the intensity of the particle beam in particles per unit area per unit time, instead of just the total number of particles,
8. the target spheres were not smooth when compared with the size of the beam particles,
9. we did not have  $m_1 \ll m_2$ , or
10. the hard spheres were distributed in a pattern.

Remember be **brief**. Just show that you know what the effect of the change is.

## 2 Rutherford Scattering

Given the same experiment as in problem 1, except with the beam particles now interacting with the target particles with a repulsive force given by  $\frac{k}{r^2}$ , where  $k$  is a constant,  $k > 0$ ,  $r$  is the distance between a target particle and a beam particle, and now the radius of the target particles,  $R$ , is negligible. So we still have  $N_{\text{tar}}$ ,  $m_1$ ,  $m_2$ ,  $u_1$ ,  $A$ , and  $N_{\text{inc}}$  as defined in problem 1.

## 2.1 Particles Deflected at Angle $\theta$

What is number of particles per unit area that go off at a deflection angle of  $\theta$  measured from the direction of the incident beam, where  $\theta = 0$  would be no scattering, and at a distance of  $D$  from the target, where  $D^2 \gg A$ .

## 2.2 Too Small Deflection Angle

In this case, with Rutherford scattering, the incident particles are interacting with the target particles over an infinitely long distance. At about what deflection angle is large enough such that we can consider the deflected particle to be scattered from just one target particle. Call this small deflection angle  $\theta_1$ . For this problem ignore electron screening effects, though for a real physical system that is very important. We never said we had electrons (or any thing physically real) in this problem.

Hint: Find the angle of deflection when the impact parameter is about half the size of the inter-target-particle spacing. If the impact parameter of a beam particle is about half the size of inter-target-particle spacing then it is probable that we will have a small scattering that is due to interactions with more than one particle. So we see that when the deflection angle is too small our two particle interaction model of scattering can no longer be used to calculate the deflection angle.