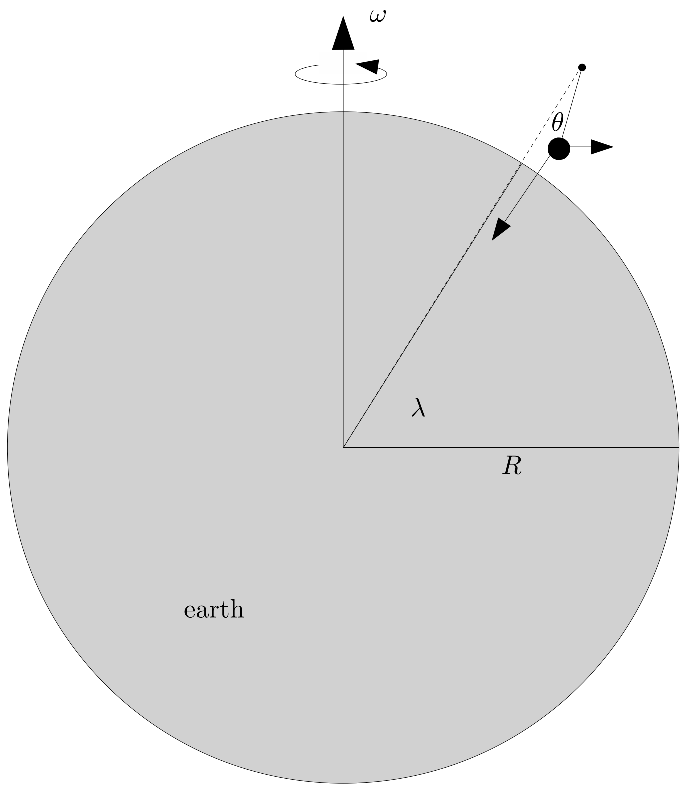
1 Small Angular Deviation from True Vertical



1.1 General Angular Deviation from True Vertical

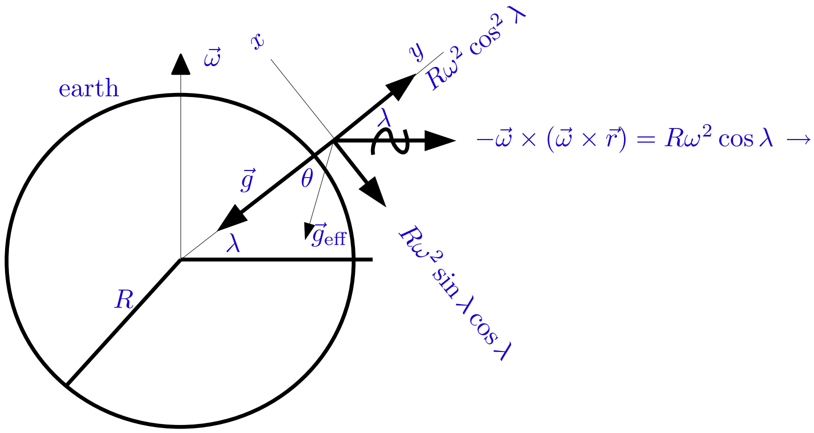
Show that the small angular deflection of a plumb line from a radially vertical line is approximately

$$\theta \approx \frac{R\omega^2 \sin\lambda \cos\lambda}{g - R\omega^2 \cos^2\lambda} \tag{1.1}$$

where R is the radius of the earth, ω is the angular frequency of rotation of the earth, g is the acceleration due to gravity at the surface of the earth, and λ is the angle of latitude (which is zero at the equator).

1.1 solution

Using the right hand rule to find the centrifugal acceleration we get



From the figure above

$$\vec{g}_{\rm eff} = \left(R\omega^2\cos^2\lambda - g\right)\hat{y} - R\omega^2\sin\lambda\cos\lambda\,\hat{x}\,. \tag{1.2}$$

So

$$\tan\theta = \frac{R\omega^2\sin\lambda\cos\lambda}{g - R\omega^2\cos^2\lambda}\,. \tag{1.3}$$

So for small θ

$$\boxed{\theta \approx \frac{R\omega^2\sin\lambda\cos\lambda}{g - R\omega^2\cos^2\lambda}}\,. \tag{1.4}$$

1.2 Maximum Angular Deviation from True Vertical

What is the maximum deflection of a plumb line measured in degrees? (Maximize by varying λ.) Use $R = 6.4\times10^6$m and $g = 9.8$m/s^2. Hint: Keep in mind what numbers are small.

1.2 solution

We can farther simplify 1.4 using $R\omega^2 \ll g$ giving

$$\theta \approx \frac{R\omega^2}{g}\sin\lambda\cos\lambda = \frac{R\omega^2}{2g}\sin 2\lambda \tag{1.5}$$

which has a maximum when

$$2\lambda = \frac{\pi}{2} \quad\Rightarrow\quad \lambda = \frac{\pi}{4}\,. \tag{1.6}$$

So

$$\theta_{\rm max} \approx \frac{R\omega^2}{2g} = \frac{6.4\times10^6{\rm m}\left(\frac{2\pi}{24\times3600{\rm s}}\right)^2}{2\times9.8{\rm m/s}^2}\,\frac{180^\circ}{2\pi\,{\rm rad}} \quad\Rightarrow\quad \boxed{\theta_{\rm max}\approx 0.099^\circ}\,. \tag{1.7}$$