1 Inertia Tensor

A rigid body is made from 8 particles all with mass m. There is one particle at each corner of a cube of side length a. Massless sticks are used to hold the particles together as a rigid body.

1.1 About a Corner of the Cube

Show that the inertia tensor for rotations about a corner of the cube, with coordinate axes along edges of the cube, is given by

$$\mathbf{I} = ma^2 \begin{bmatrix} 8 & -2 & -2 \\ -2 & 8 & -2 \\ -2 & -2 & 8 \end{bmatrix} .$$
(1.1)

1.2 About the Center of the Cube

Find the inertia tensor for rotations about the center of the cube with coordinate axes parallel to edges of the cube.

2 Inertia Tensor for a Solid Cone

A uniform solid cone has a mass M, a base radius R, and a height of h. The z axis is along the axis of symmetry of the cone. The tip of the cone is at the point of rotation which is at the origin. Show that the moment of inertia for this cone can be written as

$$\mathbf{I} = \frac{3}{20} M \begin{bmatrix} R^2 + 4h^2 & 0 & 0\\ 0 & R^2 + 4h^2 & 0\\ 0 & 0 & 2R^2 \end{bmatrix}.$$
 (2.1)