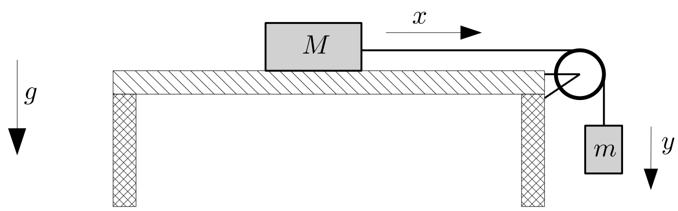
This does not cover all topics that could be on the final exam. Topics that are covered in the homework and quizzes are possible topics that may be on the final exam. Recent course topics are not covered in this homework.

1 Blocks with Pulley



A block of mass M slides on the top of a table. The coefficient of static friction between the block and the table is μ_s. The coefficient of kinetic friction between the block and the table is μ_k. The pulley is massless and frictionless. A massless stretch-less string connects the sliding block to a hanging weight. The displacement of the block is given by x. The displacement of the hanging weight is given by y.

1.1 Free Body Diagrams

Draw a free body diagram for the block and the hanging weight.

1.2 Minimum Weight Mass

What is the minimum (limiting) hanging weight mass, m_{min}, that will cause the sliding block to start to move. Answer in terms of M, and μ_s.

1.3 Acceleration

When the block is moving, what is acceleration of the sliding block, $\ddot{x}$, as a function of M, m, g, and μ_k.

2 Power Into a Simple Harmonic Oscillator

A driven damped simple harmonic oscillator consists of a mass m, connected to a spring with spring constant k, and linear damping constant b ($F_{\text{damping}} = -b\dot{x}$). A driving force acts on the mass with a force of $F(t) = F_0 \cos \omega t$ along the direction of the oscillation of mass.

2.1 Average Power In

In terms of the given parameters, find the average power $\langle P \rangle$ ($P = \vec{F} \cdot \vec{v}$) that the driving force applies to the oscillator over one cycle of the oscillator, when the oscillator is in steady state motion.

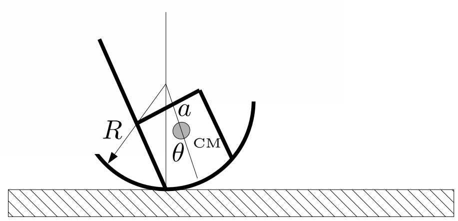
2.2 Power Resonance

Find, ω_r, the value of the driving angular frequency, ω, that maximizes this average power, $\langle P \rangle$.

3 Find a Force from an Orbit

Find the central force, $F(r)$, that allows a particle to move in a spiral orbit given by $r = k\theta$, where k is a constant.

4 Rocking Chair



A rocking chair rolls (rocks) without slipping. The radius of the rocker is R. The total mass of the rocking chair is M. The distance from the center of the rocker circle to the center-of-mass (CM) is a. The momentum-of-inertia of the rocking chair about the center-of-mass is I. No one is sitting in the rocking chair.

4.1 Lagrangian

Find the Lagrangian $L(\theta, \dot{\theta})$, where θ is the angle of rotation of the rocking chair measured from the equilibrium position.

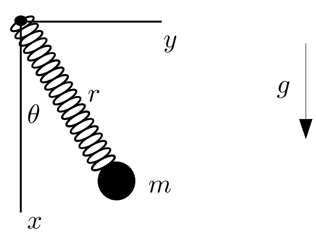
4.2 Differential Equation of Motion for θ

Find the differential equation of motion for θ. You may answer in terms of $\ddot{\theta}$, $\dot{\theta}$, θ, M, a, R, I, and g (the acceleration due to gravity).

4.3 Angular Frequency for Small Oscillations

Find the angular frequency, ω_0, for small oscillations about $\theta = 0$. You may answer in terms of M, a, R, I, and g (the acceleration due to gravity).

5 Springy Pendulum



The springy pendulum shown above has a rest spring length a, a spring constant k, and a bob mass of m. r is the distance from the origin (pivot point) to the bob. Both r and θ can change in time.

5.1 Hamiltonian

Find the generalized momentums p_r and p_θ in terms of the generalized coordinates and velocities, and find the Hamiltonian $H(p_r, r, p_\theta, \theta)$ for this system.

5.2 Differential Equations of Motion

Find the differential equations of motion for p_r, p_θ, r, and θ, in terms of $\dot{p}_r$, $\dot{p}_\theta$, $\dot{r}$, $\dot{\theta}$, p_r, p_θ, r, θ, m, k, a, and g,.

6 Inverse Rocket

A large abandoned space ship travels through space which is filled with uniformly distributed “space dust”, with mass density ρ. The only forces on the space ship are from the dust that collects on the ship as it goes through the dust. Consider the space dust to be at rest (not moving) before the ship hits it. Assume that all the dust that gets hit by the ship sticks to the ship and effectively increases the mass of the ship, and slows down the ship. All the motion is in one dimension.

6.1 Differential Equation

Let m be the mass of the ship at a given time t. Note that m is increasing in with t. Let v be the speed of ship at a given time t. v is decreasing with t.

By using conservation of momentum (or some other equivalent to Newton’s 2nd law), find the “rocket-like” equation that relates $\mathrm{d}v$, $\mathrm{d}m$, m, and v, and solve for v as a function of m.

6.2 Find $\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}\mathbf{t}}$

A is the cross-sectional area of the ship that is passing (cutting) through the dust. Find $\frac{\mathrm{d}m}{\mathrm{d}t}$ as a function of v, ρ, and A using the fact that the ship adds the mass of all the dust that the ship hits.

6.3 Solve for $\mathbf{m(t)}$ and $\mathbf{v(t)}$

The initial ship speed (at time $t = 0$) is v_0 and the initial ship mass is m_0. Using this with previous results, solve for $m(t)$ and $v(t)$ in terms of t, v_0, m_0, A and ρ.