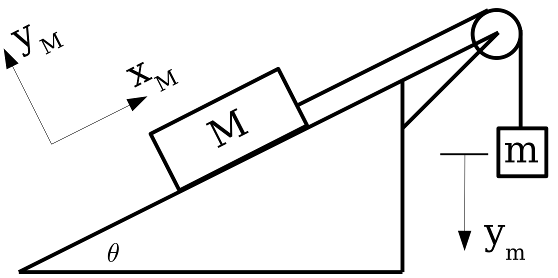
1 sliding block and weight

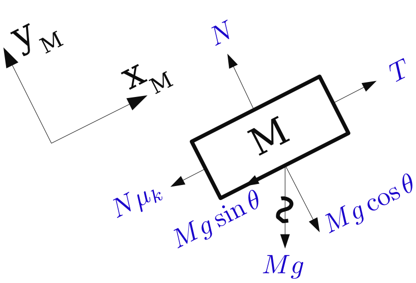
A block, with mass M, slides up a plane inclined at angle θ as measured from the horizontal. The block slides with a coefficient of kinetic friction μ_k. A massless stretch-less string connects the block to a hanging weight through a massless frictionless pulley. The mass of the hanging weight is m. Use g as the acceleration due to gravity.

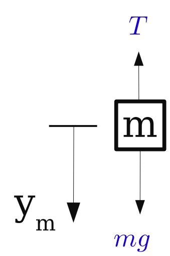


1.1 free body diagram

Draw a free body diagram the sliding block and the hanging weight. Use the coordinate systems shown in the above figure.

1.1 solution





1.2 Newton's 2nd law

From the free body diagrams write the equations of motion, from Newton's 2nd law, for the sliding block and the hanging weight. Also write an equation of constraint for the string that relates $\ddot{x}_M$ and $\ddot{y}_m$.

1.2 solution

$$\sum_M F_{y_M} = N - Mg\cos\theta = 0 \tag{1.1}$$

$$\sum_M F_{x_M} = T - N\mu_k - Mg\sin\theta = M\ddot{x}_M \tag{1.2}$$

$$\sum_m F_{y_m} = mg - T = m\ddot{y}_m \tag{1.3}$$

$$\ddot{x}_M = \ddot{y}_m \tag{1.4}$$

1.3 acceleration

Solve and simplify an expression for $\ddot{x}_M$ in terms of M, m, μ_k, θ, and g.

1.3 solution

Solving for N in equation 1.1 and plugging that into equation 1.2 gives

$$T - \mu_k M g \cos\theta - M g \sin\theta = M\ddot{x}_M. \tag{1.5}$$

Adding equation 1.3 to equation 1.5 and using 1.4 gives

$$mg - \mu_k M g \cos\theta - M g \sin\theta = M\ddot{x}_M + m\ddot{y}_m \tag{1.6}$$

$$\Rightarrow \quad mg - Mg\left(\mu_k \cos\theta + \sin\theta\right) = M\ddot{x}_M + m\ddot{x}_M. \tag{1.7}$$

Therefore $\boxed{\ddot{x}_M = \frac{mg - Mg(\mu_k \cos\theta + \sin\theta)}{m+M}}$. It's a good idea to check that this makes sense for the limiting cases $\theta \to 0$ and $\mu_k \to 0$.