# 1 potential

A particle is constrained to move in one dimension, $x$, is acted on by a force which is derived for the potential energy $U(x) = ax + \frac{b}{2}x^2$, where the constants $a$ and $b$ are positive.

## 1.1 force

What is the force, $f(x)$, from this potential as a function of $x$?

**1.1 solution**

For force and potential in 1-D, $x$,

$$f(x) = -\frac{\mathrm{d}U(x)}{\mathrm{d}x} = -\frac{\mathrm{d}}{\mathrm{d}x}\left(ax + \frac{b}{2}x^2\right) = -a - bx. \tag{1.1}$$

So

$$\boxed{f(x) = -a - bx}. \tag{1.2}$$

## 1.2 equilibrium $x$ position

Find the equilibrium position, $x_0$, the $x$ position where the force is zero.

**1.2 solution**

$$f(x_0) = 0 \quad \Rightarrow \quad 0 = -a - bx_0 \quad \Rightarrow \quad x_0 = -\frac{a}{b} \tag{1.3}$$

So

$$\boxed{x_0 = -\frac{a}{b}}. \tag{1.4}$$

## 1.3 stability

Is this equilibrium position, $x_0$, stable or unstable? Show why.

**1.3 solution**

$$\frac{\mathrm{d}^2U(x)}{\mathrm{d}x^2} = -\frac{\mathrm{d}}{\mathrm{d}x}f(x) = \frac{\mathrm{d}}{\mathrm{d}x}(a + bx) = b \tag{1.5}$$

$\frac{\mathrm{d}^2U(x_0)}{\mathrm{d}x^2} = b$ which is greater than zero therefore $x_0$ is a stable equilibrium position.