1 Hanging a Mass on a Spring

A weight, of mass $m = 200$g, is hung on a spring with unknown spring constant k. The mass is hung vertically on the spring so that gravity pulls on the mass too. The weight is released from a position where the spring is relaxed (not stretched or compressed). The weight is observed to move up and down with a period, τ, of 1.1 seconds.

Use $g = 9.8\,\frac{\text{m}}{\text{s}^2}$. Assume that all given numbers are exact. Assume there is no damping in the motion. Express your answers to at least 3 significant figures.

1.1 Spring Constant

Given these measurements, determine the spring constant of the spring, k.

1.1 solution

We define ω_0 to be the angular frequency of the motion, so $\omega_0 = \frac{2\pi}{\tau}$.

$$\omega_0^2 = \frac{k}{m} \quad \Rightarrow \quad \left(\frac{2\pi}{\tau}\right)^2 = \frac{k}{m} \quad \Rightarrow \quad k = \frac{4\pi^2 m}{\tau^2} = \frac{4\pi^2\, 200\,\text{g}\, \left(\frac{1\,\text{kg}}{1000\text{g}}\right)}{(1.1\,\text{s})^2} \approx 6.52 \frac{\text{kg}}{\text{s}^2} = 6.52 \frac{\text{N}}{\text{m}}. \tag{1.1}$$

So $\boxed{k \approx 6.52 \frac{\text{N}}{\text{m}}}$

1.2 Maximum Speed

Given these measurements, determine the maximum speed of the weight, $\dot{x}_{\text{max}}$, as it oscillates up and down.

1.2 solution

There are many ways to do this, here's at quick way: When the weight is first released the acceleration of the weight is at a maximum (in magnitude) over the period and has a value of g since the spring is not stretched. Since this is simple harmonic motion than this must be the amplitude of the acceleration, $\ddot{x}_{\text{max}} = \omega_0^2 A$, and the amplitude of the velocity must be $\dot{x}_{\text{max}} = \omega_0 A$. So we have

$$\dot{x}_{\text{max}} = \omega_0 A = \frac{A\omega_0^2}{\omega_0} = \frac{\ddot{x}_{\text{max}}}{\omega_0} = \frac{g}{\omega_0} = g\frac{\tau}{2\pi} = 9.8\,\frac{\text{m}}{\text{s}^2}\frac{1.1\,\text{s}}{2\pi} \approx 1.72\,\frac{\text{m}}{\text{s}}, \tag{1.2}$$

and so $\boxed{\dot{x}_{\text{max}} \approx 1.72\,\frac{\text{m}}{\text{s}}}$

1.3 on the Moon (10 pts extra credit)

Given the same spring, weight and initial conditions, what will be the measured period τ', and the the maximum speed of the weight $\dot{x}'_{\text{max}}$, if this experiment is performed on the surface of the moon where the acceleration due to gravity is $\frac{g}{6}$. As before, the weight is released from rest at a position where there is no stretch in the spring.

1.3 solution

The period of the oscillation is only dependent on the m and k, and these parameters do not change on the surface of the moon. So $\boxed{\tau' = 1.1\mathrm{s}}$ as it is on the earth.

$\dot{x}_{\mathrm{max}}$ is the same as before in equation 1.2, but with $\dot{x}_{\mathrm{max}}$ changed to $\dot{x}'_{\mathrm{max}}$ and $g' = \frac{g}{6}$. So with these changes to equation 1.2 we get

$$\dot{x}'_{\mathrm{max}} = g' \frac{\tau}{2\pi} = \frac{g}{6}\frac{\tau}{2\pi} = \frac{1}{6}\, 9.8\, \frac{\mathrm{m}}{\mathrm{s}^2}\frac{1.1\,\mathrm{s}}{2\pi} \approx 0.286\, \frac{\mathrm{m}}{\mathrm{s}}, \tag{1.3}$$

and so $\boxed{\dot{x}'_{\mathrm{max}} \approx 0.286 \frac{\mathrm{m}}{\mathrm{s}}}$.