1 Simple Harmonic Oscillator

A simple harmonic oscillator has the equation of motion

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + kx = 0 \tag{1.1}$$

where m and k are physical constants, x is the dependent dynamical variable (position), and t is the independent variable commonly called time. Introduce the change of (independent) variable from t to τ with

$$\tau = \omega_0 t \tag{1.2}$$

where $\omega_0^2 = \frac{k}{m}$. What is the minimum number of parameters that are needed to described this scaled version of this simple harmonic oscillator system.

1.0 solution

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}\tau}\omega_0 \quad \Rightarrow \quad \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \omega_0^2\frac{\mathrm{d}^2x}{\mathrm{d}\tau^2} \tag{1.3}$$

so with 1.1

$$m\omega_0^2\frac{\mathrm{d}^2x}{\mathrm{d}\tau^2} + kx = 0 \quad \Rightarrow \quad \frac{\mathrm{d}^2x}{\mathrm{d}\tau^2} + \frac{1}{\omega_0^2}\frac{k}{m}x = 0 \quad \Rightarrow \quad \boxed{\frac{\mathrm{d}^2x}{\mathrm{d}\tau^2} + x = 0} \tag{1.4}$$

and so that are no parameters in this scaled version of this simple harmonic oscillator system. So we have reduced the number of parameters from two to zero.