1 Toward the Moon

In this problem assume that there is no atmospheric friction, and use a static model for the earth and moon system. So this will be a pretty gross approximation. Use: the mass of the earth is M_E, the mass of the moon is M_m, and the orbit radius of the earth moon system as measured from earths center to the moons center is R_o.

1.1

Find the distance from the earths center toward the moon, R_b, where the force of gravity from the earth and the force of gravity from the moon on a small object balance (add to zero).

1.1 solution

The two gravitational forces balance giving

$$G\frac{mM_E}{R_b^2} = G\frac{mM_m}{(R_o - R_b)^2} \quad \Rightarrow \quad M_E(R_o - R_b)^2 - M_m R_b^2 = 0 \quad \Rightarrow \quad \left(1 - \frac{M_m}{M_E}\right) R_b^2 - 2R_o R_b + R_o^2 = 0. \tag{1.1}$$

Solving for R_b gives

$$R_b = \frac{2R_o \pm \sqrt{4R_o^2 - 4\left(1 - \frac{M_m}{M_E}\right) R_o^2}}{2\left(1 - \frac{M_m}{M_E}\right)} \quad \Rightarrow \quad R_b = R_o \frac{\left(1 \pm \sqrt{\frac{M_m}{M_E}}\right)}{1 - \frac{M_m}{M_E}} \tag{1.2}$$

The solution that is between the earth and the moon is

$$\boxed{R_b = R_o \frac{\left(1 - \sqrt{\frac{M_m}{M_E}}\right)}{1 - \frac{M_m}{M_E}}}. \tag{1.3}$$

Note that for the case when $M_E = M_m$, we get an indeterminate form $(\frac{0}{0})$, so we must be careful taking this limit to show that we are half way between the two bodies when the forces balance.