1 Escape Speed

In this problem assume that there is no atmospheric friction, and use a static model for the earth. You may use: the acceleration do to gravity on the surface of the earth is $g = G\frac{M_E}{R_E^2} = 9.8\text{m/s}^2$, and the raduis of the earth is $R_E = 6.38 \times 10^6\text{m}$.

1.1

Find the escape speed v_{es}– that is the initial speed when released from the surface of the earth that would allow a projectile to escape from the earth completely. Hint: Use conservation of energy, kinetic plus gravitational potential energy.

1.1 solution

We define the initial energy, E_i, as the energy of the projectile at the time of release. We define the final energy, E_f, as the energy of the projectile when it is far from the earth, and is not moving. We let the mass of the projectile be m.

$$E_i = E_f \quad \Rightarrow \quad -G\frac{mM_E}{R_E} + \frac{1}{2}mv_{\text{es}}^2 = 0 + 0 \quad \Rightarrow \quad v_{\text{es}}^2 = 2G\frac{M_E}{R_E} = 2G\frac{M_E}{R_E^2}R_E = 2gR_E \quad \Rightarrow \quad v_{\text{es}} = \sqrt{2gR_E}. \tag{1.1}$$

Plugging in numbers gives

$$v_{\text{es}} \approx \sqrt{2\ 9.8\frac{\text{m}}{\text{s}^2}\ 6.38 \times 10^6\,\text{m}} \quad \Rightarrow \quad \boxed{v_{\text{es}} \approx 1,1200\frac{\text{m}}{\text{s}}} \tag{1.2}$$