

# 1 Stationary Integral

Find  $y(t)$  and  $x(t)$  such that the following integral is stationary,

$$J = \int_{t_1}^{t_2} \left( \frac{1}{2} \dot{x}^2 + \frac{1}{2} \dot{y}^2 \right) dt, \quad (1.1)$$

where  $\dot{x} \equiv \frac{dx}{dt}$  and  $\dot{y} \equiv \frac{dy}{dt}$ .

Hint: Do so by using the Euler equations,

$$\frac{\partial f}{\partial x} - \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{x}} \right) = 0 \quad \text{and} \quad \frac{\partial f}{\partial y} - \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{y}} \right) = 0 \quad (1.2)$$

where  $f(x, \dot{x}, y, \dot{y}; t) = \frac{1}{2} \dot{x}^2 + \frac{1}{2} \dot{y}^2$ . You do not have to determine all the constants of integration.