1 Simple Harmonic Oscillator

A particle of mass m moves along that x direction under the influence of a force $f(x) = -kx$ and there is no other force.

1.1 Lagrangian

Find the Lagrangian, $L(x, \dot{x}) \equiv T - U$, in terms of m, k, x, and $\dot{x}$, for this particle.

1.1 solution

$$U = -\int -kx \, \mathrm{d}x = \frac{1}{2} k \, x^2 + C, \tag{1.1}$$

where C is a constant of integration, that we will set to zero.

$$L = T - U = \frac{1}{2} m \, v^2 - \frac{1}{2} k \, x^2 = \frac{1}{2} m \, \dot{x}^2 - \frac{1}{2} k \, x^2, \tag{1.2}$$

$$\Rightarrow \quad \boxed{L\left(x, \dot{x}\right) = \frac{1}{2} m \, \dot{x}^2 - \frac{1}{2} k \, x^2}. \tag{1.3}$$

1.2 Equation of Motion

Apply Lagranges equations to this Lagrangian to get the equations of motion for this particle. Your answer should be like $\ddot{x} = ?$.

1.2 solution

Lagranges equations for this system is

$$\frac{\partial L}{\partial x} - \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{x}} = 0 \quad \Rightarrow \quad -k \, x - \frac{\mathrm{d}}{\mathrm{d}t} \left(m \dot{x}\right) = 0 \quad \Rightarrow \quad m \ddot{x} = -k \, x \quad \Rightarrow \quad \boxed{\ddot{x} = -\frac{k}{m} \, x}. \tag{1.4}$$