1 Spinning a Mass on a Spring

A particle, of mass m, is connected to a spring, with spring constant k. The spring has a rest length a. One end of the spring is held fixed while the particle rotates around the fixed end at a constant angular frequency ω, as shown in the figure below. The spring stays straight and connected to the particle, spinning with it about the fixed end of the spring. Let r be the position of the particle measured from the point of rotation, so the Lagrangian will be a function of r, $\dot{r}$, and maybe t, time ($L(r, \dot{r}, t)$). Using the Lagrangian method, find the equation of motion for r ($\ddot{r} = ?$).