1 Angular Momentum

Consider two particles, one with mass m_1 and position given by $\vec{r}_1$, and the other with mass m_2 and position given by $\vec{r}_2$. $\vec{r}_1$ and $\vec{r}_2$ are both measured from the same coordinate system. The center of mass position of the two particles is,

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}\,. \tag{1.1}$$

If we measure the positions $\vec{r}_1$ and $\vec{r}_2$ from the position of the center of mass then

$$0 = m_1\vec{r}_1 + m_2\vec{r}_2\,. \tag{1.2}$$

The angular momentum of the two particles is

$$\vec{L} = m_1\vec{r}_1 \times \dot{\vec{r}}_1 + m_2\vec{r}_2 \times \dot{\vec{r}}_2\,. \tag{1.3}$$

Show that the angular momentum of the two particles as measured about the center of mass position is given by

$$\vec{L} = \mu\,\vec{r}\times\dot{\vec{r}}, \quad \text{where} \quad \mu \equiv \frac{m_1 m_2}{m_1 + m_2}\,, \quad \text{and} \quad \vec{r} \equiv \vec{r}_1 - \vec{r}_2\,. \tag{1.4}$$