

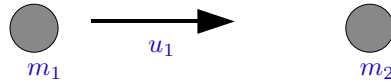
1 Inelastic Collision

A particle of mass m_1 and velocity u_1 collides with a particle of mass m_2 which was at rest. The two particles stick together. What fraction of the original kinetic energy is lost in the collision? Simplify your result.

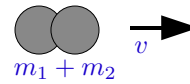
Hint: What is $\frac{T_i - T_f}{T_i}$? Momentum is conserved.

1.0 solution

Before Collision:



After Collision:



Conservation of linear momentum gives

$$m_1 u_1 = (m_1 + m_2) v \quad \Rightarrow \quad v = \frac{m_1}{m_1 + m_2} u_1. \quad (1.1)$$

The fraction of the original kinetic energy that is lost is

$$\frac{T_i - T_f}{T_i} = \frac{\frac{1}{2} m_1 u_1^2 - \frac{1}{2} (m_1 + m_2) v^2}{\frac{1}{2} m_1 u_1^2}. \quad (1.2)$$

Plugging in v from equation 1.1 gives

$$\begin{aligned} \frac{T_i - T_f}{T_i} &= \frac{m_1 u_1^2 - (m_1 + m_2) \left(\frac{m_1}{m_1 + m_2} \right)^2 u_1^2}{m_1 u_1^2} = \frac{m_1 - \frac{m_1^2}{m_1 + m_2}}{m_1} \\ &= 1 - \frac{m_1}{m_1 + m_2} = \frac{(m_1 + m_2) - m_1}{m_1 + m_2} \quad \Rightarrow \quad \boxed{\frac{T_i - T_f}{T_i} = \frac{m_2}{m_1 + m_2}}. \end{aligned} \quad (1.3)$$