1 Rocket Speed at Maximum Momentum

The rocket starts at rest with initial mass m_0 in outer space (no gravity). The rocket propels its self by expelling mass at a constant rate of $-\dot{m} = \alpha$ and with a relative exhaust speed of u. What is the speed of the rocket when it has its maximum momentum, v', as a function of m_0, u, and α?

Hint: You do not need to solve any differential equations. You can do this by manipulating the free space rocket equation $m\,\dot{v} = -u\,\dot{m}$.

1.0 solution

$$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = \dot{m}v + m\dot{v} = -\alpha v + m\dot{v} = -\alpha v + (-u\dot{m}) = -\alpha v - u\,(-\alpha) = \alpha\,(u - v) \tag{1.1}$$

where we used $\dot{m} = -\alpha$, and the rocket equation $m\dot{v} = -u\dot{m}$. We wish to maximize mv in time where m is the mass of the rocket, v is the speed of the rocket, and $v = v'$ is the speed of the rocket that maximizes mv, so

$$\frac{\mathrm{d}}{\mathrm{d}t}(mv)\bigg|_{v=v'} = 0 \quad \Rightarrow \quad \alpha\,(u - v') = 0 \quad \Rightarrow \quad \boxed{v' = u}. \tag{1.2}$$