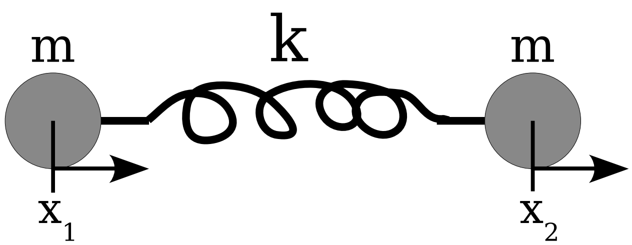
1. Write your name at the top of each paper that you hand in.
2. There are 4 problems. Points for each part varies. Partial credit may be given only if your work is clearly shown.
3. Start each problem on a new page of paper.
4. This will be due in class at 9:30 AM, Tuesday, Feb. 15.
5. All that you hand in must be your own work (Honor Code).
6. Label all work for each problem subsection and box all problem subsection answers.

1 (20 points) Two Masses and a Spring



The two masses, both with mass m, are connected by a spring with spring constant k. The positions of the masses are given by the generalized coordinate variables x_1 and x_2 respectively. $x_1 = 0$ and $x_2 = 0$ is a resting equilibrium position.

1.1 (12) Normal Modes

Find the normal-angular frequencies (squared), ${\omega_i}^2, i = 1, 2$, and the normal-amplitude vectors in terms of the generalized coordinate variable basis vector $\vec{q} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. One mode is a rigid body mode and has a angular frequency of zero. Express your answer in terms of m, and k.

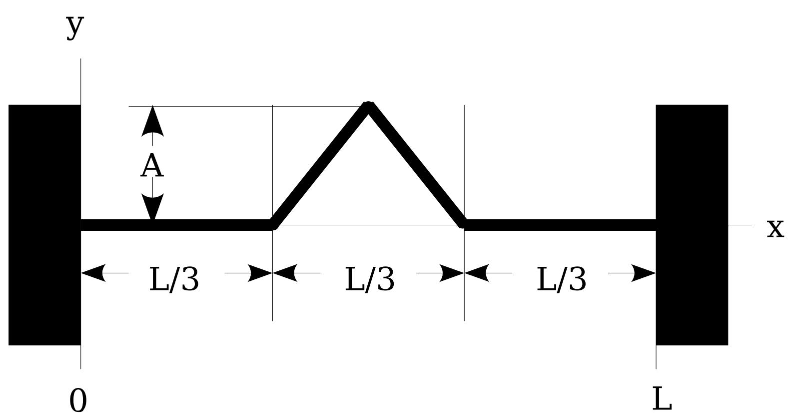
1.2 (8) Particular Solution

At time $t = 0$ the left mass is displaced to $x_1 = b$ and the right mass is at $x_2 = 0$. Both masses are initially not moving.

Find the position of the masses, x_1 and x_2, as a function of time, t.

2 (20 points) Waves on a String

This problem is most easily done using wave construction pictures. If you use this method, for your solution present clear drawings of your wave constructions. Lined paper may help. Messy drawings may not be accepted.



2.1 (10) Wave Profile at a Given Time

The figure above shows the shape of the string at time $t = 0$. The string starts at rest at time $t = 0$. The speed of the wave on this string is v, and of course the size of the disturbance $A \ll L$, so the x-scale is not the same as the y-scale in the above picture.

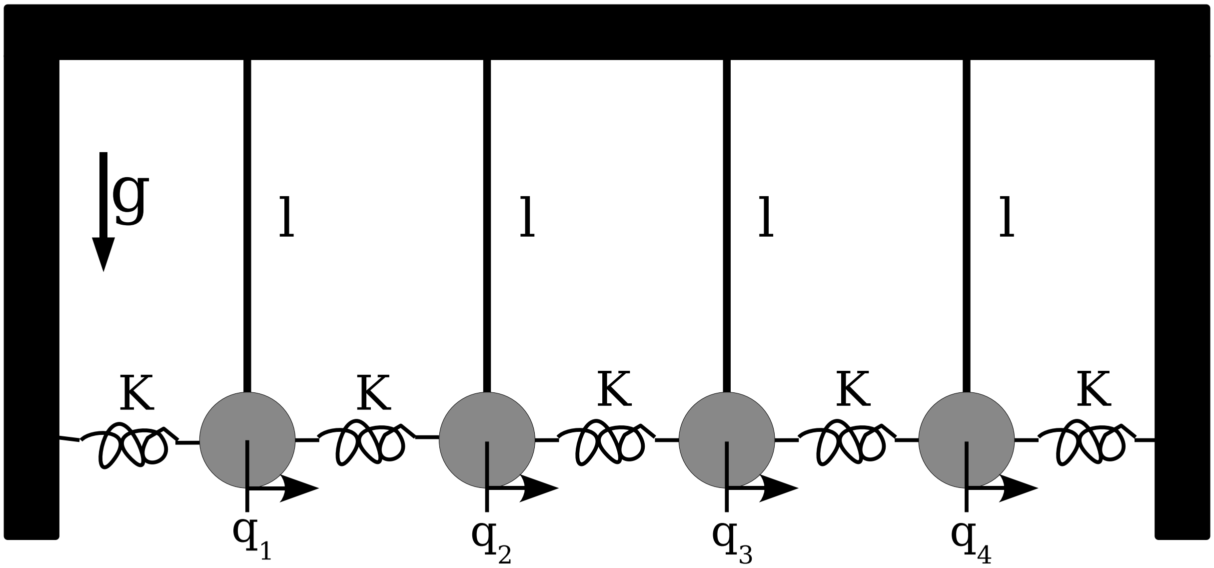
Draw sketches of the string at times $t = \frac{L}{4v}$, $t = \frac{L}{2v}$, $t = \frac{L}{v}$, and $t = \frac{2L}{v}$.

2.2 (10) Moving to Start With

The figure above shows the shape of the string at time $t = 0$. Now the string starts out with the triangular pulse moving to the right at time $t = 0$. The speed of the wave on this string is v, and of course the size of the disturbance $A \ll L$.

Draw sketches of the string at times $t = \frac{L}{4v}$, $t = \frac{L}{2v}$, $t = \frac{L}{v}$, and $t = \frac{2L}{v}$.

3 (30 points) Array of Pendulums and Springs



This figure shows four masses hanging from four pendulums with springs between all the masses and springs connecting the two end masses to a fixed point. The equilibrium distance between all masses is the same. All the spring constants are K. All the masses have mass m. All the pendulums have a length l. The pendulums all hang along the vertical at equilibrium. The swing of the pendulums is small. The masses only move small amounts in the plus or minus q directions shown in the figure. This system has longitudinal modes of oscillation. We will start by considering the case of any number of masses, n, and then solve a particular problem with four masses.

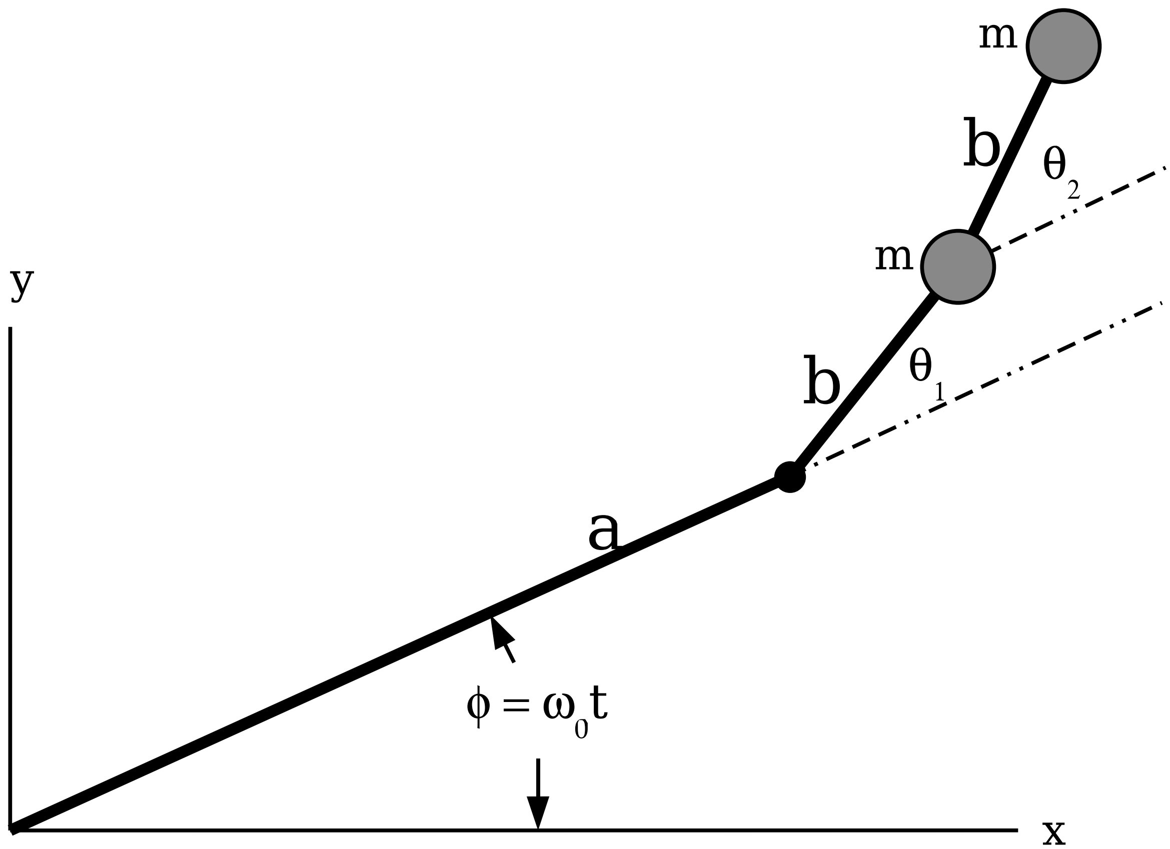
3.1 (18) Normal Modes

Find expressions for the normal mode frequencies, ω_N, and amplitudes, $\vec{a}_N$, for the case when there are n masses, where n is any integer (not just 4). Use the same notation as in Fowles section 11.5. Let k be the particle number and N be the mode number.

3.2 (12) Particular Solution

Now consider that there are four masses. Find an expression for the displacements, $\vec{q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}$, as a function of time, t, given that at $t = 0$, $q_1(0) = B$, $q_2(0) = q_3(0) = 0$, $q_4(0) = -B$, and none of them are moving initially.

4 (30 points) Spinning Double Pendulum



A stick that spins at a fixed rate of ω_0. At a distance a for the point at which the stick is rotating is a pivot at which a double pendulum is pivoting. The angle θ_1 is measured relative to a line along the stick of length a. The angle θ_2 is measured relative to a line that passes through the the second pendulum pivot point and is parallel to the stick of length a. The length of each of the two pendulums in the double pendulum is b. The mass of both masses in the two pendulums is m.

All the motion is in a horizontal plane. There is no gravity.

4.1 (10) Lagrangian

In terms of m, a, b, ω_0, θ_1, and θ_2, write an expression for the Lagrangian of this system using θ_1, and θ_2 as your generalize coordinate variables. Do not assume that anything is small in this part. Give the exact Lagrangian of this system.

4.2 (8) Equations of Motion

You may have noticed that the Lagrangian does not easily lend it's self into the standard quadratic forms, that Fowles speaks of, to study normal modes. But you know that from your experience, when you where a child spinning this thing over your head, that the two masses tend to a dynamic equilibrium position with positions $\theta_1 = 0$ and $\theta_2 = 0$. So we'll find the equations of motion and hope that gives us a form that we can see normal small oscillations with.

In terms of m, a, b, ω_0, θ_1, and θ_2, write an expression for the equations motion for this system and expand it about the $\theta_1 = 0$ and $\theta_2 = 0$, the dynamic equilibrium position.

If your expression for the Lagrangian is not very close to correct, you may not get credit for propagation of error in this part.

4.3 (4) M and K Matrices

From your equations of motion find the **M** and **K** matrices, that Fowles speaks of, for this system. Answer using m, a, b, and ω_0, with a general coordinate vector defined by $\vec{q} = \begin{pmatrix} \text{amount of } \theta_1 \\ \text{amount of } \theta_2 \end{pmatrix}$. You may divide out any common factors, that do not effect the motion, like for example m.

4.4 (8) Normal Modes

Find the normal-angular frequencies (squared), ${\omega_N}^2, N = 1, 2$, and the normal-amplitude vectors, $\vec{a}_N$. Express the normal-amplitude vectors, $\vec{a}_N$, in same coordinate variable basis vectors that you used in section 4.3. You may leave your expression for the normal-amplitude vectors, $\vec{a}_N$, as functions of ${\omega_N}^2$.